GE/Tempo
RM 61TMP
-32

63-3-5

TEMPO

THREE-DIMENSIONAL RAY TRACE COMPUTER PROGRAM FOR ELECTROMAGNETIC WAVE PROPAGATION STUDIES

RM 61TMP-32
DASA 1232

GENERAL ELECTRIC

$12.50

(4) $12.50
(5) 851 750

(6) THREE-DIMENSIONAL RAY TRACE COMPUTER PROGRAM
FOR ELECTROMAGNETIC WAVE PROPAGATION STUDIES,

(7) NA
(8) NA
(9) NA
(12) 170p.
(13) NA
(16) NA
(17) NA
(20) U
(21) NA

(14) RM 61TMP-32
(18) DASA (19) 1232

(10) by Walter F. Dudziak

Submitted to: Defense Atomic Support Agency
Washington 25, D. C.

U.C

(15) Under Contract: DA-49-146-XZ-038

(11) 1 May 1961

TECHNICAL MILITARY PLANNING OPERATION
GENERAL ELECTRIC COMPANY
SANTA BARBARA, CALIFORNIA

ABSTRACT

A computer program is described for use on the IBM-704/7090 electronic data processing machine or any large computer accepting FORTRAN. The necessary modifications for use on these two computers are simple and self-evident. The computer program permits the computation of detailed ray patterns portraying the spatial distribution of rays emitted from a transmitter whose geographic coordinates with respect to the center of the earth are known. This program deals with the solution of the differential equations, given by Hamiltonian optics, for ray paths in non-isotropic, three-dimensionally nonhomogeneous media whose complex phase refractive index is given by the Appleton-Hartree formula.

~~This report is to be considered as a first attempt~~ in presenting an account of the current status of the development of this program, which has yielded many useful results,* ~~Presented also are~~ sample calculations as well as some results that ~~have been~~ were obtained by using this program are also presented.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

SECTION I
INTRODUCTION

What is customarily referred to as reflection in electromagnetic wave propagation, is actually the result of an integrated effect of a phenomenon of refraction (i. e., the gradual changing of the direction of the electromagnetic energy transport vector). Insofar as the phenomenon of refraction is concerned it is well established that the spatial gradient of electron density plays a crucial role in controlling the propagation behavior of an electromagnetic wave. Although this is well known, propagation studies so frequently incorporate a curious mixture of simple refraction phenomena (in the use of Snell's law) and of mirror-like reflection. Hence, in these studies they omit an accounting of these spatial gradients of electron density as well as their variation. As generally known, these oversimplified studies derive from Snell's law the condition for reflection, that being, that at the spatial point of reflection, the electron density must attain the value of $1.24 \times 10^4 f^2 \cos^2 I$ electron/cc, where f is the propagation frequency in Mc/sec and I is the angle of incidence of the ray upon the first layer of the assumed stratified ionosphere. Such a simplified approach is necessary because of the difficult task that leads to a numerical solution of a propagation problem which incorporates these electron density gradients in its solution.

With increasing use of transmitters in satellites, as well as, for the understanding of the behavior of wave propagation under severly abnormal atmospheric conditions, it becomes important to take a realistic account of the spatial gradient of the electron density in electromagnetic wave propagation studies. Only in this manner will it become possible to usefully utilize the new satellite propagation techniques in studies designed toward the understanding of the atmospheric ionization-deionization phenomena and through this the detailed structure of the ionosphere.

Within the last ten years some effort has been made in constructing analogue computers for the study of ray propagation which accounted for spatial electron density gradients (as for example D. F. Hartree, et al., Manchester, England; M. S. Wong, AFCRL, Bedford, Mass.).

Some of these analogue computers were and still are limited to spatial electron density gradients in a particular direction thus forcing the propagation problem into a two-dimensional plane, or to the study of the behavior of refraction on wave propagation. These constraints are built into the analogue computer and are not easily changed.

One approach that avoids these constraints is the use of a large electronic data processing system where the ordered logical flow controlling any calculation, is easily varied. Combining such a programmed computer with Hamiltonian optics, which give the desired ray tracing equations for a nonhomogeneous, non-isotropic mediu, and the Appleton-Hartree formula for the complex refractive indes, permits in addition to three dimensional ray tracing, the simultaneous study of numerous other variables of the propagation problem. Such an approach to the ray trace propagation problem is presented in its present state of development. A great deal of improvement in some of the routines is possible. As a result, the writer would like to encourage correspondence concerning these matters. In addition, it should be stated, that a computer program for solving the three-dimensional ray trace problem has also been written for the Ferranti Mercury Computer at Manchester University by C. B. Haselgrove and J. Haselgrove.[2]

SECTION II
COMPUTATIONAL PROCEDURE

A. RAY TRACE EQUATIONS

When electromagnetic waves are propagated through a medium whose dielectric constant or index of refraction is a varying function of the path, the waves undergo a change in direction, or refractive bending, and a retardation in the velocity of propagation. The spatial relationship expressing this angular bending of a ray of an electromagnetic wave can be determined by basing the theory of rays and waves on a variational principle (Fermat's) in space. By a ray is meant the path travelled by the transport vector of electromagnetic wave energy between the transmitter and an associated electromagnetic field-intensity point in space. This Hamilton Theory starts from the variational principle $\delta \int \mu ds = 0$, where μ is a medium function or index of refraction, depending on position and direction. From this principle the theory constructs the properties of systems of rays and the waves associated with them (i.e., extremals and transversals, in the language of the calculus of variations). Because of the stationarity in time, the theory may be regarded as a statical one, the rays being fixed curves in space and the waves fixed surfaces. Neither wave-length nor frequency is involved. Likewise the waves form a continuous set of surfaces, not distinguished as crests and troughs. This theory, whether in the form preferred by Hamilton or otherwise, has been the subject of many books under the general title "Geometric Optics".

Thus, applying Hamiltonian optics leads to the general Hamilton's Equations[1] for ray paths of electromagnetic waves in a three-dimensional non-isotropic nonhomogeneous medium. From them Haselgrove[2] has derived the following set of equations for ray paths in a spherical coordinate system in a format suitable for numerical integration on high speed computers:

$$\frac{dr}{d\tau} = \frac{1}{\mu^2}\left(\sigma_r - \mu \frac{\partial \mu}{\partial \sigma_r} \right) \qquad (1)$$

$$\frac{d\theta}{d\tau} = \frac{1}{r\mu^2}\left(\sigma_\theta - \mu\frac{\partial\mu}{\partial\sigma_\theta}\right) \tag{2}$$

$$\frac{d\varphi}{d\tau} = \frac{1}{\mu^2 r \sin\theta}\left(\sigma_\varphi - \mu\frac{\partial\mu}{\partial\sigma_\varphi}\right) \tag{3}$$

$$\frac{d\sigma_r}{d\tau} = \frac{1}{\mu}\frac{\partial\mu}{\partial r} + \sigma_\theta\frac{d\theta}{d\tau} + \sin\theta\,\sigma_\varphi\frac{d\varphi}{d\tau} \tag{4}$$

$$\frac{d\sigma_\theta}{d\tau} = \frac{1}{r}\left[\frac{1}{\mu}\frac{\partial\mu}{\partial\theta} - \sigma_\theta\frac{dr}{d\tau} + r\cos\theta\,\sigma_\varphi\frac{d\varphi}{d\tau}\right] \tag{5}$$

$$\frac{d\sigma_\varphi}{d\tau} = \frac{1}{r\sin\theta}\left[\frac{1}{\mu}\frac{\partial\mu}{\partial\varphi} - \sin\theta\,\sigma_\varphi\frac{dr}{d\tau} - r\cos\theta\,\sigma_\varphi\frac{d\theta}{d\tau}\right] \tag{6}$$

In these equations r, θ, and φ are the spatial coordinates of a spherical system; μ is the arbitrary index of refraction; $\vec{\sigma}$ is a vector directed normal to the phase fronts of the ray of magnitude μ with σ_r θ, and σ_φ its respective components in the r, θ, and φ directions; τ is the time of phase travel along the ray path (i.e., $f\Delta\tau/c$ = the number of wavelengths in the medium along the ray path, where f is the electromagnetic wave frequency and c the velocity of light in vacuum).

It is noteworthy that the partial derivatives of μ appear explicitly in Equations 1 to 6, in accordance with the fact that the gradients of μ play crucial roles in determining the spatial ray paths.

This closed set of first-order partial differential equations which will describe the propagation behavior of an electromagnetic wave

under geometric optics conditions, can be integrated simultaneously by a point-by-point numerical process if expressions can be developed for the necessary derivatives of the phase refractive index μ. The quantity μ and its derivatives are obtained by using the Appleton-Hartree formula[3] as the definition of the complex phase refractive index M.

The derivatives are derived under the conditions of ray optics, that is, that the imaginary part of M^2 is very much smaller than the real part. As an aid for computer use and comparison with published works of others[2,4], the Appleton-Hartree formula is written as:

$$M^2 = (\mu - j\kappa)^2 = 1 - \frac{2X(1 - X - jZ)}{D} \tag{7}$$

$$D = 2(1 - X - jZ)(1 - jZ) - Y^2 \sin^2 \psi + S \tag{8}$$

$$S = \pm\left[(Y \sin\psi)^4 + 4Y^2(1 - X - jZ)^2 \cos^2 \psi \right]^{1/2} \tag{9}$$

where

M = the complex phase refractive index

X = a scalar quantity representing the normalized electron density

$$\frac{4\pi Ne^2}{m\omega^2} = \frac{\omega_p^2}{\omega^2}$$

ω_p = plasma frequency at the spatial point

ω = angular wave frequency = $2\pi f$

m, e = mass and charge of an electron

N = electron density at a spatial point

Y = normalized magnitude of the earth's magnetic field vector $\vec{Y}$ =

$$\frac{e\vec{H}}{mc\omega} = \frac{\vec{\omega}_c}{\omega}$$

ω_c = magnitude of the gyromagnetic frequency of the electron about the earth's magnetic field

Z = normalized collision frequency = (ν/ω)

ν = collision frequency at a spatial point in collisions per second

ψ = angle defined by the inner product of the vectors $\vec{\sigma}$ and $\vec{Y}$ =

$$\cos^{-1}\left[\frac{\sigma_r Y_r + \sigma_\theta Y_\theta + \sigma_\varphi Y_\varphi)}{(\mu Y)}\right]$$

κ = $\frac{ck}{\omega}$ = imaginary part of the complex phase refractive index

c = velocity of light in vacuum

k = absorption coefficient of the wave per unit length of path (it is proportional to the conductivity of the medium)

It is noted that there are two possible values for the complex index of refraction M corresponding to the plus and minus sign on S which represent two different modes of ionospheric propagation. These are commonly called "ordinary" and "extraordinary" modes for the plus and minus sign respectively. Also, the Appleton-Hartree formula (Equations 7 to 9) is notable in that it presents μ, which actually is a spatial function of six variables, in the form containing purely algebraic operations on factors or terms each of which is a function of at most three variables, that is, either of r, θ, φ or σ_r, σ_θ, σ_φ. This reduces the representation of μ to a numerical problem, easily solvable to current computer techniques.

If i represents any one of the spatial spherical coordinates r, θ, and φ, then the partial derivatives of μ with respect to the components of the wave normal, $\vec{\sigma}$, can be easily shown to be

$$\frac{\partial\mu}{\partial\sigma_i} = \frac{\partial\mu}{\partial\psi}\frac{\partial\psi}{\partial\sigma_i} = \frac{\partial\mu}{\partial\psi}\left(\frac{\sigma_i\, Y\cos\psi - \sigma\, Y_i}{\sigma^2 Y\sin\psi}\right) \tag{10}$$

This useful transformation also enjoys the following property:

$$\text{When } \psi \to o,\quad \frac{\partial\mu}{\partial\psi} \to o,\ \frac{\partial\psi}{\partial\sigma_i} \to \infty \qquad \text{but } \frac{\partial\mu}{\partial\sigma_i} \to o \tag{11}$$

To evaluate $\partial\mu/\partial\sigma_i$; the necessary partial derivative is:

$$\frac{\partial\mu}{\partial\psi} = \mathrm{Re}\,\frac{\partial M}{\partial\psi} = \mathrm{Re}\left\{\frac{(M^2-1)\,(Y^2\sin\psi\cos\psi)}{MD}\left[1 - \frac{1}{S}\left[(Y\sin\psi)^2 - 2(1-X-jZ)^2\right]\right]\right\}$$

$$\therefore\ \frac{\partial\mu}{\partial\psi} = \left\{- Y^2\sin\psi\cos\psi\left[a_o(a_2a_5-b_2b_5) - b_o(a_2b_5+b_2a_5)\right]\right\} \tag{12}$$

where a_o, b_o and all following a_j, b_j are defined in the List of Useful Formulae. The partial derivatives of the real part of the phase refractive index with respect to the spatial coordinates ($i = r$, θ, or φ) are similarly obtained by use of the relationship

$$\frac{\partial\mu}{\partial i} = \frac{\partial\mu}{\partial X}\frac{\partial X}{\partial i} + \frac{\partial\mu}{\partial Y}\frac{\partial Y}{\partial i} + \frac{\partial\mu}{\partial Z}\frac{\partial Z}{\partial i} + \frac{\partial\mu}{\partial\psi}\frac{\partial\psi}{\partial i} \tag{13}$$

where

$$\frac{\partial \mu}{\partial X} = \mathrm{Re}\frac{\partial M}{\partial X} = \mathrm{Re}\left\{\frac{1}{MD}\left[2X - 1 + jZ + (M^2 - 1)\left(1 - jZ + \frac{2Y^2(1-X-jZ)\cos^2\psi}{S}\right)\right]\right\}$$

$$\therefore \frac{\partial \mu}{\partial X} = \left\{a_o\left[(2X-1) - (a_4a_5 - b_4b_5)\right] + b_o\left[Z + a_5b_4 + b_5a_4\right]\right\} \qquad (14)$$

$$\frac{\partial \mu}{\partial Y} = \mathrm{Re}\frac{\partial M}{\partial Y} = \mathrm{Re}\left\{\frac{(M^2-1)}{MD} Y\left[(\sin\psi)^2 - \frac{1}{S}[Y^2\sin^4\psi + 2(1-X-jZ)^2\cos^2\psi]\right]\right\}$$

$$\therefore \frac{\partial \mu}{\partial Y} = Y\left\{(a_oa_5 - b_ob_5)\left[a_6 - (\sin\ \ )^2\right] - b_6(a_ob_5 + b_oa_5)\right\} \qquad (15)$$

$$\frac{\partial \mu}{\partial Z} = \mathrm{Re}\frac{\partial M}{\partial Z} = -\,\mathrm{Im}\left\{\frac{1}{MD}\left[X + (M^2-1)\left(2 - X - 2jZ + \frac{2Y^2(1-X-jZ)}{S}\cos^2\psi\right)\right]\right\}$$

$$\frac{\partial \mu}{\partial Z} = \left[b_o(X - a_5a_7 + b_5b_7) - a_o(b_5a_7 + a_5b_7)\right] \qquad (16)$$

and where Re and Im represent, respectively, the real and imaginary part of the complex expression. The partial derivative of the angle ψ (which is the angle defined by the inner product of the normalized geomagnetic field vector $\vec{Y}$ and the wave normal vector $\vec{\sigma}$), with respect to the spatial coordinates r, θ, and φ, measures the change in spatial direction of the earth's magnetic field since the calculation is made holding $\vec{\sigma}$ constant. The partial derivatives $\partial X/\partial i$, $\partial Y/\partial i$ and $\partial Z/\partial i$ are obtainable from the analytical expressions for the spatial variation of the electron density, geomagnetic field and collision frequency. Examples of these will be considered later.

In addition to Equations 1 to 6, which define the spatial ray path, it is usually desirable to calculate the optical path length s, the time of travel T, as well as, the one way absorption A, of the energy of an electromagnetic pulse. The equation describing the differential optical path is given by

$$\frac{ds}{d\tau} = \frac{1}{\mu^2}\left[\mu^2 + \left(\frac{\partial\mu}{\partial\Psi}\right)^2\right]^{1/2} \tag{17}$$

In determining the time of travel T, a distinction must be made between two electromagnetic wave velocities. The phase velocity, $v_p = c/\mu$, is defined as the spatial velocity with which a point of constant phase moves. Group velocity, $v_g = d\omega/d\,(\omega/v_p)$, is the spatial velocity of electromagnetic energy travel; put into other words, it is the velocity of a "Maxwell Demon" who remains at the same point on the envelope of the advancing wave. From these two definitions it can be easily shown that the time (in seconds) of energy pulse travel can be written as:

$$\frac{dT}{d\tau} = \frac{1}{c}\left[1 + \frac{\omega}{\mu}\frac{\partial\mu}{\partial\omega}\right]; \tag{18}$$

where

$$\frac{\partial\mu}{\partial\omega} = \mathrm{Re}\ \frac{\partial M}{\partial\omega} =$$

$$-\ \mathrm{Re}\left\{\frac{1}{MD\omega}\left[X(2X + jZ) + (M^2-1)\left[2-2jZ-jZX + \frac{2(Y\cos\Psi)^2}{S}(1-X-jZ)(1+X)\right]\right]\right\}$$

$$\therefore \frac{\partial\mu}{\partial\omega} = -\frac{1}{\omega}\left[a_o(2X^2 - a_5a_8 + b_5b_8) + b_o(XZ + b_5a_8 + b_8a_5)\right] \tag{19}$$

The one-way absorption, A (in nepers), suffered by the energy of an electromagnetic pulse is determined by

$$\frac{dA}{d\tau} = - \frac{\omega}{c} \frac{\kappa}{\mu} A = - \frac{k}{\mu} A \qquad (20)$$

where k (which is proportional to the spatial conductivity) is the absorption of the wave per unit length of path.

The solution of this set of first order partial differential equations will describe the propagation characteristics of an electromagnetic wave in a heterogeneous anisotropic medium.

B. LIST OF USEFUL FORMULAE FOR RAY TRACING

As an aid for the computer solution of these differential equations the following list of formulae are found to be very useful. As before, Re and Im respectively represent the real and imaginary parts of the complex quantity.

$$\mathrm{Re}S = S_1 = R_S \cos \theta_S \tag{21}$$

$$\mathrm{Im}S = S_2 = R_S \sin \theta_S \tag{22}$$

$$R_S = \left\{ \left[(Y \sin\psi)^4 + (2Y \cos\psi)^2 \left[(1 - X)^2 - Z^2 \right] \right]^2 + \left[(2Y \cos\psi)^2 \left[2(1 - X)Z \right] \right]^2 \right\}^{1/4} \tag{23}$$

$$\theta_S = \frac{1}{2} \tan^{-1} \left\{ \frac{(2Y \cos\psi)^2 \left[2(X - 1)Z \right]}{(Y \sin\psi)^4 + (2Y \cos\psi)^2 \left[(1 - X)^2 - Z^2 \right]} \right\} \tag{24}$$

$$\mathrm{Re}D = d_1 = \left\{ 2 \left[(1 - X) - Z^2 \right] - (Y \sin\psi)^2 + S_1 \right\} \tag{25a}$$

$$\mathrm{Im}D = d_2 = \left[S_2 - 2Z(2 - X) \right] \tag{25b}$$

$$\mathrm{Re}M = m_1 = \mu = R_m \cos \theta_m \tag{26}$$

$$\mathrm{Im}M = m_2 = -\kappa = R_m \sin \theta_m \tag{27}$$

$$R_m \left\{ \left(1 - \frac{2X\left[(1 - X)d_1 - Zd_2\right]}{d_1^{\ 2} + d_2^{\ 2}}\right)^2 + \left(\frac{2X\left[Zd_1 + (1 - X)d_2\right]}{d_1^{\ 2} + d_2^{\ 2}}\right)^2 \right\}^{1/4} \tag{28}$$

$$\theta_m = \frac{1}{2} \tan^{-1} \left\{ \frac{2X\left[Zd_1 + (1 - X)d_2\right]}{d_1^{\ 2} + d_2^{\ 2} - 2X\left[(1 - X)d_1 - Zd_2\right]} \right\} \tag{29}$$

$$a_o = \frac{(m_1d_1 - m_2d_2)}{(m_1d_1 - m_2d_2)^2 + (m_1d_2 + m_2d_1)^2} \tag{30}$$

$$b_o = \frac{(m_1d_2 + m_2d_1)}{(m_1d_1 - m_2d_2)^2 + (m_1d_2 + m_2d_1)^2} \tag{31}$$

$$a_1 = \left\{ 2\left[(1 - X)^2 - Z^2\right] - (Y \sin \psi)^2 \right\} \tag{32}$$

$$b_1 = 4Z(1 - X) \tag{33}$$

$$a_2 = \left[1 + \frac{(a_1S_1 - b_1S_2)}{S_1^{\ 2} + S_2^{\ 2}} \right] \tag{34}$$

$$b_2 = \left[\frac{S_1 b_1 + a_1 S_2}{S_1^{\,2} + S_2^{\,2}} \right] \tag{35}$$

$$a_4 = \left\{ 1 + \frac{2(Y\cos\psi)^2}{S_1^{\,2} + S_2^{\,2}} \left[S_1(1 - X) - ZS_2 \right] \right\} \tag{36}$$

$$b_4 = \left\{ Z + \frac{2(Y\cos\psi)^2}{S_1^{\,2} + S_2^{\,2}} \left[S_2(1 - X) + ZS_1 \right] \right\} \tag{37}$$

$$a_5 = \frac{2X\left[(1 - X)d_1 - Zd_2\right]}{d_1^{\,2} + d_2^{\,2}} = (1 + m_2^{\,2} - m_1^{\,2}) \tag{38}$$

$$b_5 = \frac{2X\left[Zd_1 + (1 - X)d_2\right]}{d_1^{\,2} + d_2^{\,2}} = 2m_2 \tag{39}$$

$$a_6 = \frac{S_1 \left\{ (Y\sin^2\psi)^2 + 2\cos^2\psi \left[(1 - X)^2 - Z^2 \right] \right\} - S_2 \left[(2\cos\psi)^2 Z(1 - X) \right]}{S_1^{\,2} + S_2^{\,2}} \tag{40}$$

$$b_6 = \frac{S_2 \left\{ (Y\sin^2\psi)^2 + 2\cos^2\psi \left[(1 - X)^2 - Z^2 \right] \right\} + S_1 \left[(2\cos\psi)^2 Z(1 - X) \right]}{S_1^{\,2} + S_2^{\,2}} \tag{41}$$

$$a_7 = \left\{ (2 - X) + \frac{2(Y \cos\psi)^2}{S_1^2 + S_2^2} \left[S_1(1 - X) - S_2 Z \right] \right\} \tag{42}$$

$$b_7 = \left\{ 2Z + \frac{2(Y \cos\psi)^2}{S_1^2 + S_2^2} \left[S_1 Z + S_2(1 - X) \right] \right\} \tag{43}$$

$$a_8 = 2 \left\{ 1 + \frac{(Y \cos\psi)^2}{S_1^2 + S_2^2} \left[(1 - X^2) S_1 - S_2 Z(1 + X) \right] \right\} \tag{44}$$

$$b_8 = \left\{ Z(2 + X) + \frac{2(Y \cos\psi)^2}{S_1^2 + S_2^2} \left[S_1 Z(1 + X) + S_2(1 - X^2) \right] \right\} \tag{45}$$

Nomenclature Used in Ray Trace Equations

r, θ, φ	spatial coordinates of a spherical system
$\vec{\sigma}$	wave normal or refractive index vector
$\sigma_r, \sigma_\theta, \sigma_\varphi$	vector components ($\vec{\sigma}$)
τ	time of phase travel (in units of length)
μ	real part of complex phase refractive index
f	electromagnetic wave frequency
c	velocity of light in vacuum
m, e	mass and charge of an electron
Re	real part of the complex expression
Im	imaginary part of the complex expression
M	complex phase refractive index
X	scalar quantity representing normalized electron density
ω_p	plasma frequency at the spatial point
ω	angular wave frequency
N	electron density at spatial point
Y	normalized magnitude of the earth's magnetic field vector $\vec{Y}$
ω_c	magnitude of the gyromagnetic frequency of the electron about the earth's magnetic field
Z	normalized collision frequency (ν/ω)
ν	collision frequency at spatial point in collisions/sec.
ψ	angle defined by inner product of vectors $\vec{\sigma}$ and $\vec{Y}$

κ	imaginary part of the complex phase refractive index
k	absorption coefficient of the wave per unit length of path (proportional to the conductivity of the medium)
i	represents each of the spatial spherical coordinates r, θ, φ
T	time of travel (seconds)
s	optical path length (km)
A	one-way absorption (nepers)
v_p	spatial velocity with which a point of constant phase moves
v_g	group velocity - spatial velocity of electromagnetic energy travel

C. COORDINATE TRANSFORMATION

If one takes three orthogonal planes intersecting at a point, one knows that the position of any point S in space is uniquely determined by the three perpendiculars from S on these planes, each with its proper sign. However, the problem of selecting the most useful orientation of such an orthogonal system is difficult since the usefulness of a coordinate system partially depends on the problem definition and the application of its solution. This ray trace program is designed as a sub-set of a much larger computer programming effort[7] where the earth's geomagnetic field plays an important part. To minimize the number of computer transformations in the design of the over-all program, an earth centered spherical coordinate system (r, θ, φ) was chosen, whose z-component is coincident with the magnetic dipole axis.

This selection permits the application of the computer program to a great many studies of ray path problems because it accounts for the earth curvature and accepts for solution any electromagnetic radiating source whose transmitter location specifications of elevation, E, and azimuth, A, angles, as well as, geographic latitude, φ_R, geographic longitude, λ_R, and position with respect to the surface of the earth are known. Because the usefulness of this computer program can be extended by modification to other coordinate systems, as for example, an earth centered geographic system, or a radar coordinate system, the necessary coordinate transformations from the radar to the earth centered geomagnetic coordinate system will be described in detail.

For the discussion of this coordinate transformation, it is assumed that the electromagnetic wave transmitter is earth bound (i.e., fixed to the surface of the earth) at a geographic latitude φ_R and a geographic longitude λ_R. It is assumed that the radar is positioned so that the transmitting direction is described by the elevation angle, E, with respect to the tangent plane to the earth surface at the radar location, and an azimuth angle, A, measured from the radar coordinate that is tangent to a great circle passing through the north geographic pole. The azimuth angle is plus when measured counter-clockwise from the coordinate axis, ζ, whose positive direction points in the direction of geographic north. It is further assumed that S is a spatial point on the non-deviated portion of the ray a distance, R, from the electromagnetic wave transmitter. This is the spatial starting

point at which the numerical methods necessary for solution of the differential equations, must be initialized.

A transformation is required from the spherical radar coordinate system to the magnetic coordinate system whose origin is at the center of the earth.

The necessary matrices which are required to transform R, A, E coordinates to r, θ, φ coordinates can be arrived at by a series of simple matrix transformations.

1. Let ε, η, ζ be a set of orthogonal coordinates with origin on the surface of the earth. Let ε-axis be perpendicular to the earth's surface while ζ is directed (geographically) northward and η to the east. As stated above R is the slant range; E is the elevation angle; and A is the azimuth angle. Hence going from R, E, A to ε, η, ζ

$$\begin{Bmatrix} \varepsilon \\ \eta \\ \zeta \end{Bmatrix} = \begin{Bmatrix} R \sin E \\ R \cos E \sin A \\ R \cos E \cos A \end{Bmatrix} \tag{46}$$

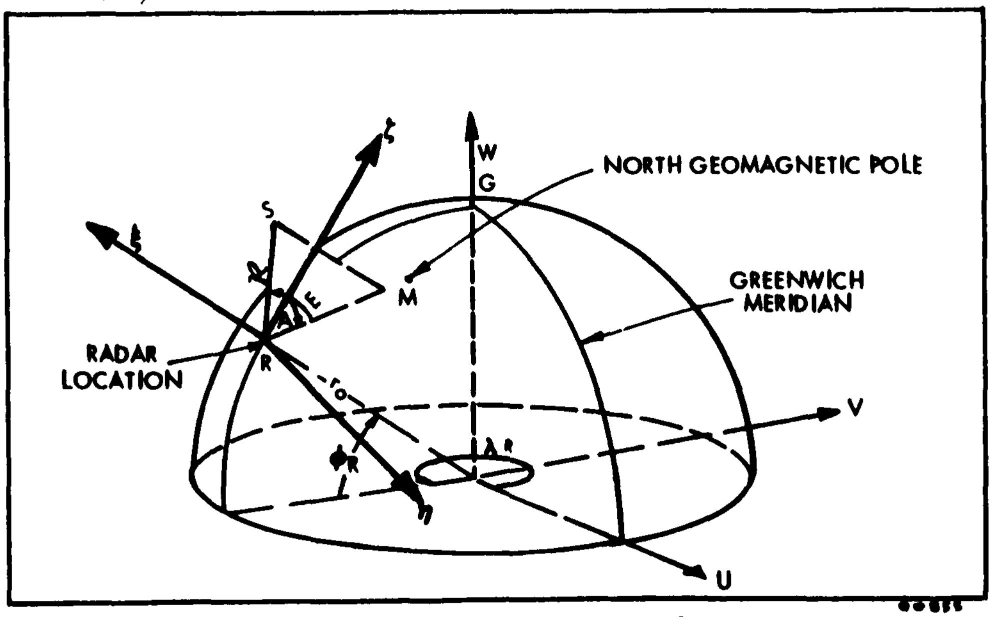


Figure 1a. Geometry of Coordinate Transformation

2. Let x_1, y_1, z_1 equal an orthogonal coordinate system with origin on the earth's axis of rotation. The x_1-axis is in the latitude plane of the radar site and passes through the radar site. The z_1-axis is coincident with the north geographic coordinate w. A translation and rotation is required in going from ε, η, ζ to x_1, y_1, z_1.

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \cos\varphi_R & 0 & -\sin\varphi_R \\ 0 & 1 & 0 \\ \sin\varphi_R & 0 & \cos\varphi_R \end{pmatrix} \begin{pmatrix} \varepsilon \\ \eta \\ \zeta \end{pmatrix} + \begin{pmatrix} r_o \cos\varphi_R \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = (b_{ij}) \begin{pmatrix} \varepsilon \\ \eta \\ \zeta \end{pmatrix} + (b_i) \tag{47}$$

where r_o equals the earth's radius.

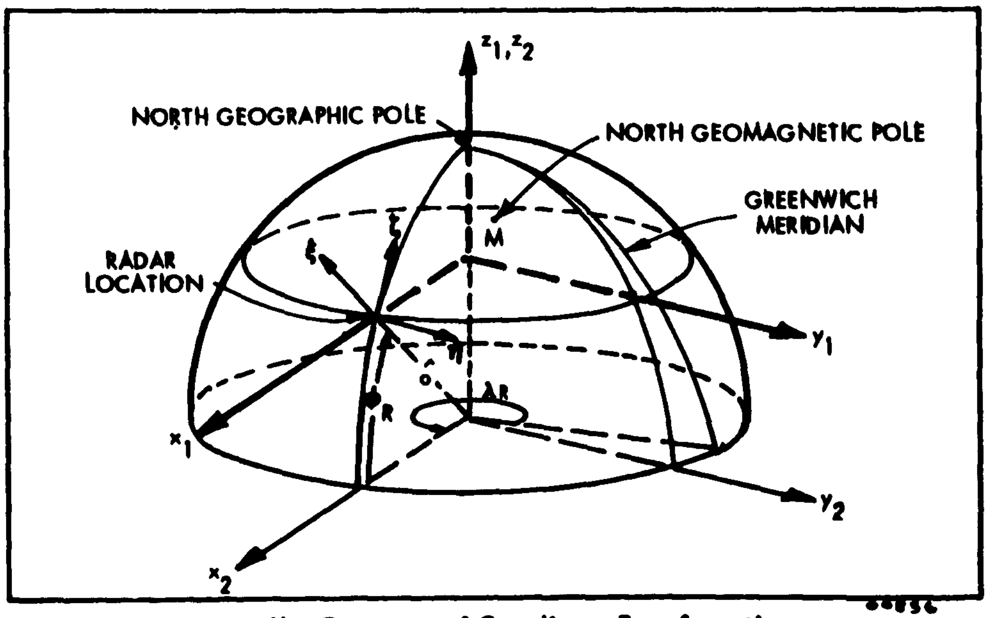


Figure 1b. Geometry of Coordinate Transformation

3. Let x_2, y_2, z_2 equal the orthogonal coordinate system with origin at the earth's center. Let the x_2-axis be parallel to the x_1-axis and z_2 coincide with w, hence also with z_1. Then going from x_1, y_1, z_1 to x_2, y_2, z_2 by translation

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ r_o \sin\varphi_R \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + (c_i) \tag{48}$$

4. Let x_3, y_3, z_3 represent an orthogonal coordinate system with origin at the earth's center such that the x_3-axis intersects the zero degree longitudinal geomagnetic meridian while the z_3-axis coincides with w. Hence going from x_2, y_2, z_2 to x_3, y_3, z_3 by rotation about the z_2-axis yields,

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} \cos(\lambda_M - \lambda_R) & \sin(\lambda_M - \lambda_R) & 0 \\ -\sin(\lambda_M - \lambda_R) & \cos(\lambda_M - \lambda_R) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = (d_{ij}) \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \tag{49}$$

where as before λ_M and φ_M represent the geographic longitude and latitude of the geomagnetic north pole M.

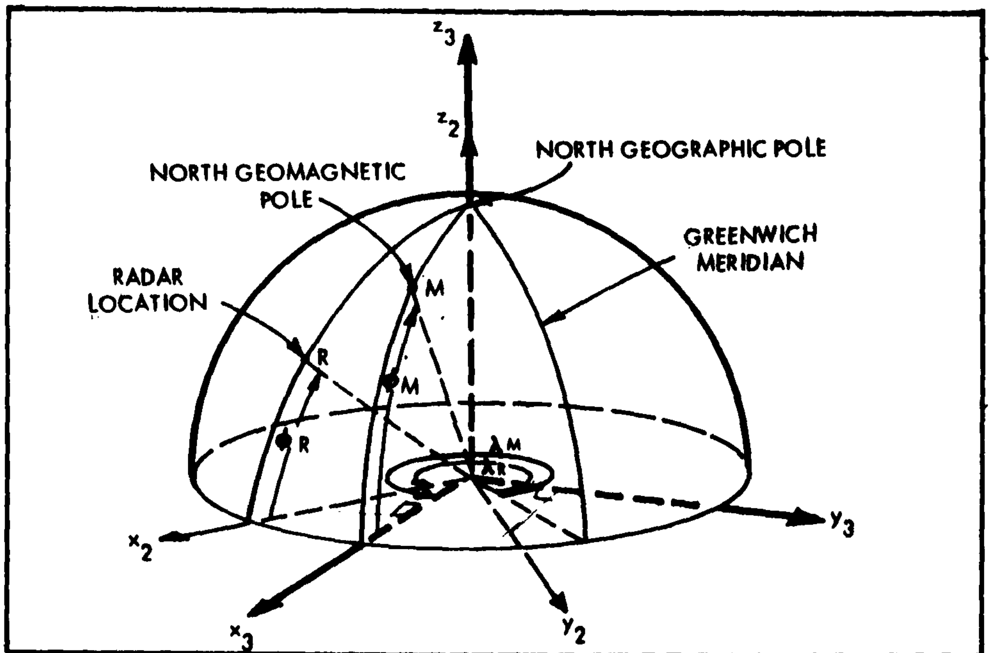


Figure 1c. Geometry of Coordinate Transformation

5. Let x, y, z represent an orthogonal coordinate system with origin at the earth's center. Let the x-axis pass through the great circle connecting the geographic and geomagnetic poles while the z-axis passes through the geomagnetic pole M. In going from x_3, y_3, z_3 to x, y, z by a rotation one obtains

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sin\varphi_M & 0 & -\cos\varphi_M \\ 0 & 1 & 0 \\ \cos\varphi_M & 0 & \sin\varphi_M \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (e_{ij}) \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} \tag{50}$$

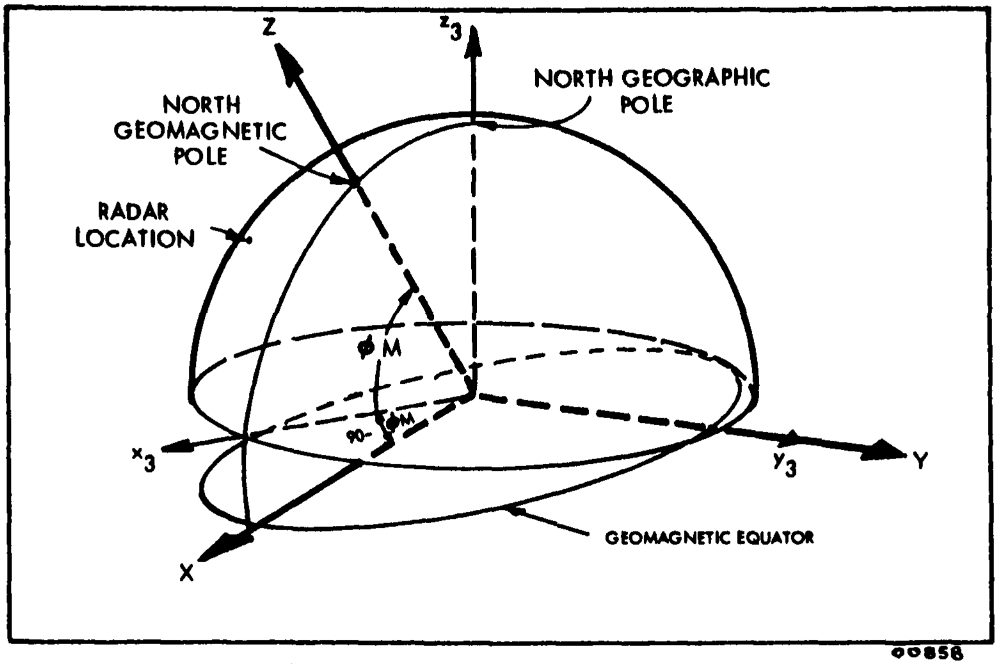


Figure 1d. Geometry of Coordinate Transformation

Hence by matrix multiplication one can transform from radar coordinates R, E, A to the earth centered coordinates x, y, z where the magnetic dipole axis of the earth coincides with the z-axis. From this coordinate system one can simply transform to the desired spherical coordinate system r, θ, φ.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (e_{ij}) \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = (e_{ij})\,(d_{ij}) \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = (e_{ij})\,(d_{ij}) \left\{ \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + (c_i) \right\}$$

$$= (e_{ij})\,(d_{ij}) \left\{ (b_{ij}) \begin{pmatrix} \varepsilon \\ \eta \\ \zeta \end{pmatrix} + (b_i + c_i) \right\} \tag{51}$$

$$(g_{ij}) = (e_{ij})\,(d_{ij})\,(b_{ij}) = (f_{ij})\,(b_{ij}) \tag{52}$$

$$(f_{ij}) = \begin{pmatrix} \sin\varphi_M \cos(\lambda_M - \lambda_R) & \sin\varphi_M \sin(\lambda_M - \lambda_R) & -\cos\varphi_M \\ -\sin(\lambda_M - \lambda_R) & \cos(\lambda_M - \lambda_R) & 0 \\ \cos\varphi_M \cos(\lambda_M - \lambda_R) & \cos\varphi_M \sin(\lambda_M - \lambda_R) & \sin\varphi_M \end{pmatrix} \tag{53}$$

$$g_{ij} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix} \tag{54}$$

$$g_{11} = \left|\cos\varphi_R \sin\varphi_M \cos(\lambda_M - \lambda_R) - \cos\varphi_M \sin\varphi_R\right|$$

$$g_{12} = \sin\varphi_M \sin(\lambda_M - \lambda_R)$$

$$g_{13} = \left|-\sin\varphi_R \sin\varphi_M \cos(\lambda_M - \lambda_R) - \cos\varphi_R \cos\varphi_M\right|$$

$$g_{21} = -\sin(\lambda_M - \lambda_R) \cos\varphi_R$$

$$g_{22} = \cos(\lambda_M - \lambda_R)$$

$$g_{23} = \sin\varphi_R \sin(\lambda_M - \lambda_R)$$

$$g_{31} = \left|\cos\varphi_R \cos\varphi_M \cos(\lambda_M - \lambda_R) + \sin\varphi_M \sin\varphi_R\right|$$

$$g_{32} = \cos\varphi_M \sin(\lambda_M - \lambda_R)$$

$$g_{33} = \left|-\sin\varphi_R \cos\varphi_M \cos(\lambda_M - \lambda_R) + \sin\varphi_M \cos\varphi_R\right|$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R\,(g_{ij}) \begin{pmatrix} \sin E \\ \cos E \sin A \\ \cos E \cos A \end{pmatrix} + r_o\,(g_{i1}) = r \begin{pmatrix} \sin\theta \cos\varphi \\ \sin\theta \sin\varphi \\ \cos\theta \end{pmatrix} \tag{55}$$

By use of these matrix transformations the Cartesian coordinates, (x, y, z), and from them the spherical coordinates, (r, θ, φ), of the earth centered geomagnetic coordinate system can be determined for the spatial starting point S and the earth bound transmitter R. These can be expressed as:

$$\begin{aligned} x_S = {} & \left[\cos(\lambda_M - \lambda_R) \sin\varphi_M \cos\varphi_R - \cos\varphi_M \sin\varphi_R\right] (R \sin E + r_o) \\ & + \left[\sin(\lambda_M - \lambda_R) \sin\varphi_M\right] R \cos E \sin A \\ & - \left[\cos(\lambda_M - \lambda_R) \sin\varphi_M \sin\varphi_R + \cos\varphi_M \cos\varphi_R\right] R \cos E \cos A \end{aligned} \tag{56}$$

$$\begin{aligned} y_S = {} & \left[- \sin(\lambda_M - \lambda_R) \cos\varphi_R\right] (R \sin E + r_o) + \left[\cos(\lambda_M - \lambda_R)\right] R \cos E \sin A \\ & + \left[\sin(\lambda_M - \lambda_R) \sin\varphi_R\right] R \cos E \cos A \end{aligned} \tag{57}$$

$$\begin{aligned} z_S = {} & \left[\cos(\lambda_M - \lambda_R) \cos\varphi_M \cos\varphi_R + \sin\varphi_M \sin\varphi_R\right] (R \sin E + r_o) \\ & + \left[\sin(\lambda_M - \lambda_R) \cos\varphi_M\right] R \cos E \sin A \\ & - \left[\cos(\lambda_M - \lambda_R) \cos\varphi_M \sin\varphi_R - \sin\varphi_M \cos\varphi_R\right] R \cos E \cos A \end{aligned} \tag{58}$$

When R = 0, that is, for a point on the surface of the earth,

$$x_R = r_o \left[\cos(\lambda_M - \lambda_R) \sin\varphi_M \cos\varphi_R - \cos\varphi_M \sin\varphi_R\right] \quad (59)$$

$$y_R = - r_o \sin(\lambda_M - \lambda_R) \cos\varphi_R \quad (60)$$

$$z_R = r_o \left[\cos(\lambda_M - \lambda_R)\cos\varphi_M \cos\varphi_R + \sin\varphi_M \sin\varphi_R\right] \quad (61)$$

From simple trigonometric considerations (Figure 2a) it can be shown that the radar slant range, R, measured from the transmitter to the spatial starting point S is given by

$$R = - r_o \sin E + \sqrt{(r_o + h_S)^2 - r_o^{\,2} \cos^2 E} \quad (62)$$

where h_S is the vertical height of the starting point above its projection, (point P) on the surface of the earth.

Equations 1 through 6 point out that in addition to these transformations, the components of the directed normal to the phase fronts, $\vec{\sigma}$, at the starting point S are to be determined in this coordinate system. From spherical and plane trigonometric considerations, (Figure 2), it can be shown that these components are given by

$$\sigma_r = \sigma \cos e \quad (63)$$

$$\sigma_\theta = \sigma \sin e \cos a \quad (64)$$

$$\sigma_\varphi = - \sigma \sin e \sin a \quad (65)$$

Angle e can be evaluated directly by employing the law of sines. This yields

$$e = \sin^{-1}\left(\frac{r_o \cos E}{r_o + h_S}\right) \tag{66}$$

Angle a is the geomagnetic bearing angle (Figure 2) measured positive in a clockwise direction from geomagnetic north. By use of spherical trigonometry it is expressible by

$$a = \tan^{-1}\left[\frac{\sin(\Phi_S - \Phi_R)\sin\theta_R}{\cos\theta_S \sin\theta_R \cos(\Phi_S - \Phi_R) - \sin\theta_S \cos\theta_R}\right] \tag{67}$$

where Φ_R, θ_R, and Φ_S, θ_S are the geomagnetic longitudes and colatitudes, respectively, of the radar transmitter, R, and the spatial starting point, S. The geomagnetic angles are obtained from the following expressions

$$\Phi_R = \tan^{-1}\left(\frac{y_R}{x_R}\right) = \tan^{-1}\left(\frac{g_{21}}{g_{11}}\right)$$

$$= \tan^{-1}\left[\frac{-\sin(\lambda_M - \lambda_R)\cos\varphi_R}{\cos(\lambda_M - \lambda_R)\sin\varphi_M \cos\varphi_R - \cos\varphi_M \sin\varphi_R}\right] \tag{68}$$

$$\theta_R = \cos^{-1}\left(\frac{z_R}{r_R}\right) = \cos^{-1}(g_{31})$$

$$= \cos^{-1}\left[\cos(\lambda_M - \lambda_R)\cos\varphi_M \cos\varphi_R + \sin\varphi_M \sin\varphi_R\right] \tag{69}$$

$$\Phi_S = \tan^{-1}\left(\frac{y_S}{x_S}\right) \tag{70}$$

$$\theta_S = \cos^{-1} \frac{z_S}{r_o + h_S} \tag{71}$$

One additional useful expression can be obtained from these algebraic relations. The parameter is the angle Ψ at the spatial starting point S which is defined by the inner product of the magnetic field vector and the wave normal. By the application of the sine and cosine laws to the geometry of Figure 2-a, it can be shown that

$$\Psi = \cos^{-1}\left[-(\cos e \sin I + \sin e \cos I \cos a)\right] \tag{72}$$

where angle I is the magnetic inclination angle. The inclination angle[9] is only a function of the geomagnetic latitude at the particular point in question.

$$I = \tan^{-1}\left[2 \operatorname{cotan} \theta_S\right] \tag{73}$$

Expressions arising from the inverse coordinate transformation, that is, transformation from the geomagnetic coordinates (r, θ, φ) to the radar coordinates R, E, A can be easily developed from these formulae. Although used in the computer program they will not be presented here.

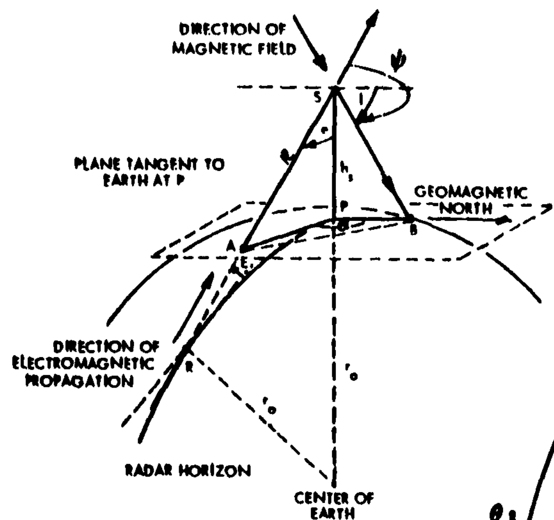


Figure 2a. Exaggerated Geometric Configuration for Starting Point Values

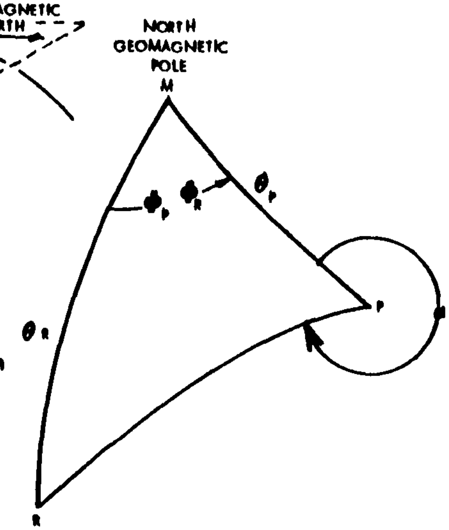


Figure 2b. Spherical Triangle Illustrating the Geomagnetic Bearing Angle

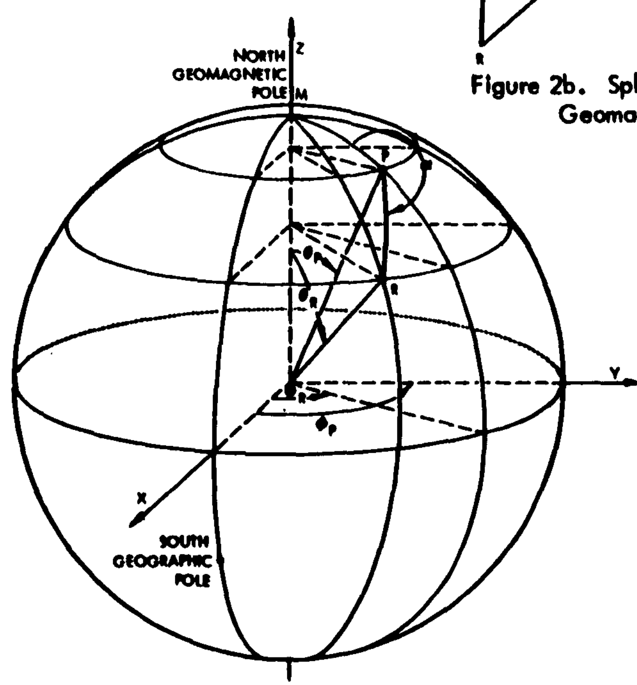


Figure 2c. Geomagnetic Coordinate System

Figure 2. Geometry for Starting Point and Geomagnetic Coordinate System

Nomenclature Used in Coordinate Transformation

r, θ, φ	spatial point in an earth centered spherical co-ordinate system
E	radar elevation angle
A	radar azimuth angle
$\varphi_{R, M}$	geographic latitude of point R or M, respectively
$\lambda_{R, M}$	geographic longitude of point R or M, respectively
S	spatial starting point on nondeviated portion of ray
R	distance from electromagnetic wave transmitter to starting point
ε, η, ζ	set of orthogonal coordinates with origin on the surface of the earth at radar site
x_1, y_1, z_1	orthogonal coordinate system with origin on the earth's axis of rotation
x_2, y_2, z_2	orthogonal coordinate system with origin at earth's center
x_3, y_3, z_3	orthogonal coordinate system with origin at earth's center
w	z component of the geographic coordinate system (u, v, w)
h_S	height of starting point above its projection on the surface of the earth
r_o	radius of earth
α	geomagnetic bearing angle
Φ_R, Φ_S	geomagnetic longitudes of points R and S

θ_R, θ_S	geomagnetic co-latitudes of points R and S
$\sigma_r, \sigma_\theta, \sigma_\varphi$	physical components of a vector of length μ, that is directed normal to the phase front
ψ	angle between magnetic field vector and the wave normal
I	angle of magnetic inclination

D. MODEL IONOSPHERE

As shown under Computational Procedure, the refractive index M and its spatial derivatives are dependent on the normalized density, X, and its spatial gradients, $\partial X/\partial r$, $\partial X/\partial \theta$, and $\partial X/\partial \varphi$. For an evaluation of these quantities an analytic model ionosphere can be chosen. One such ionospheric model that was found useful, is a spherical electron distribution as measured from a point in space. A reason for its selection is presented under Computational Results. Other uses as well as other ionospheric models are covered elsewhere[7, 11].

Let r_b, θ_b, φ_b represent the spatial location, B, of the center of the selected spherical ionosphere in the geomagnetic spherical coordinate system (r, θ, φ). The values of these coordinates are obtainable from the specified geographic latitude, longitude and height above the earth surface of point B, in an analogous procedure as described for the transformation of coordinates of the spatial starting point S. Then the electron density at a spatial point r, θ, φ for an assumed spherical ionosphere can be written as:

$$N(r, \theta, \varphi, r_b, \theta_b, \varphi_b) = \frac{A}{\mathcal{R}^n} \tag{74}$$

where

$$\mathcal{R} = \left\{ (r_b \sin\theta_b \cos\varphi_b - r\sin\theta\cos\varphi)^2 + (r_b \sin\theta_b \sin\varphi_b - r\sin\theta\sin\varphi)^2 + (r_b\cos\theta_b - r\cos\theta)^2 \right\}^{1/2} = \left\{ XP^2 + YP^2 + ZP^2 \right\}^{1/2} \tag{75}$$

For this discussion A and n are appropriately chosen constants which give the desired electron density N (electrons/cc). By use of Equations 74 and 75, it is easily shown that the spatial electron density gradients can be expressed in the following manner,

$$\frac{\partial N}{\partial r} = \frac{nA}{\ell^{n+2}} \left[XP \sin\theta \cos\varphi + YP \sin\theta \sin\varphi + ZP \cos\theta \right] \tag{76}$$

$$\frac{\partial N}{\partial \theta} = \frac{nAr}{\ell^{n+2}} \left[XP \cos\theta \cos\varphi + YP \cos\theta \sin\varphi - ZP \sin\theta \right] \tag{77}$$

$$\frac{\partial N}{\partial \varphi} = \frac{nAr}{\ell^{n+2}} \left[- XP \sin\theta \sin\varphi + YP \sin\theta \cos\varphi \right] \tag{78}$$

Some results obtained by use of such a spherical ionospheric model will be discussed later.

E. MODEL OF EARTH'S MAGNETIC FIELD

Because magneto-ionic effects on the propagation of electromagnetic waves through an ionized medium are taken into account in the derivation of the equations under Computational Procedure, it is necessary to specify the normalized external magnetic field of the earth, $\vec{Y}$, its components Y_r, Y_θ, Y_φ and its spatial derivatives $\partial Y/\partial r$, $\partial Y/\partial \theta$, $\partial Y/\partial \varphi$. It is known that the earth's magnetic field can be approximated by an earth centered magnetic dipole with its axis displaced such that the geographic longitude $\lambda_M = 70.1°W$ and the geographic latitude $\Phi_M = 78.6°N$. The magnetic potential, V, at a distant point from such a dipole is related to the magnetic moment, $\mathcal{M}$, by the expression

$$V(r,\theta) = -\frac{\mathcal{M}\cos\theta}{r^2} = -\frac{(Y_e r_o^3)\cos\theta}{r^2} \tag{79}$$

where r, θ are the geomagnetic coordinates of the spatial point irrespective of the coordinate φ and as before, r_o = radius of the earth. In this equation Y_e is the magnitude of the normalized magnetic field at the earth's surface on the magnetic equator. By use of this algebraic equation all the desired quantities can be derived. They are

$$Y = Y_e \left(\frac{r_o}{r}\right)^3 (1 + 3\cos^2\theta)^{1/2} \tag{80}$$

where, as previously defined, Y is the normalized magnitude of the earth's magnetic field vector $\vec{Y} = (eH/mc\omega) = \omega_c/\omega$

$$Y_r = 2\,Y_e \left(\frac{r_o}{r}\right)^3 \cos\theta = \frac{Y}{\sqrt{1 + \frac{1}{4}\tan^2\theta}} \tag{81}$$

$$Y_\theta = Y_e \left(\frac{r_o}{r}\right)^3 \sin\theta = \frac{1}{2}\,Y_r \tan\theta \tag{82}$$

$$Y_\varphi = \frac{\partial Y}{\partial \varphi} = 0 \tag{83}$$

$$\frac{\partial Y}{\partial r} = -\frac{3Y}{r} \tag{84}$$

$$\frac{\partial Y}{\partial \theta} = -\frac{3Y \sin\theta \cos\theta}{\left[1 + 3\cos^2\theta\right]} \tag{85}$$

F. MODEL OF ATMOSPHERIC COLLISION FREQUENCY

For some of the trial calculations the atmospheric collision frequency was found from assumed exponential variations of collision frequency with height. The atmosphere was radially stratified and an approximate exponential equation was curve-fitted to measured experimental data for each stratified region. Hence, for each region the following relations were used to obtain Z and $\partial Z/\partial r$, $\partial Z/\partial\theta$, $\partial Z/\partial\varphi$ that are required by the ray trace equations.

$$Z = \frac{\nu}{\omega} = ae^{-b(r - r_o)} \tag{86}$$

$$\frac{\partial Z}{\partial r} = -bZ \tag{87}$$

$$\frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial \varphi} = 0 \tag{88}$$

The dependence of collision frequency on a localized temperature distribution and degree of ionization[7] complicates these simple relations. These complications (as derived by D. Archer) as well as their effects will not be discussed at this time.

G. COMPUTATIONAL RESULTS

The preceding equations are only a summary of the required set which will permit the detailed calculation of a ray path in three-dimensional space. Because of this, it becomes clear that the only realistic approach to the solution of this problem is the utilization of computer techniques, otherwise, the welter of data that must be handled through use of numerical methods, is beyond effective human handling capacity. However, the development of a computer program which can perform countless number of calculations, poses the very difficult task of determining the correctness of a computed result. To simplify this "debugging" task the classical idea of elastic collision between charged particles was borrowed from nuclear physics. It has been shown[10] that if the electron density falls off as the inverse square of the distance from the center of a spherical electron distribution ($N = A/\mathscr{R}^2$), various exact expressions can be obtained, since the ray equations at zero azimuth are integrable.

The geometry of such a distribution, as well as, three ray paths computed by use of the computer program are shown in Figure 3. In the figure the center of the sphere is located by the fixed coordinates (R_o, β_o) with respect to the radar. The derivations are made in two dimensions, hence only the two-dimensional coordinate system (ξ, ζ) is used. The ray path has an initial elevation angle E_1 and its distance of closest approach to the center of the refracting sphere is denoted by $\mathscr{R}_o$. The coordinates of any point on the ray path with respect to the center of the sphere are ($\mathscr{R}$, β) and with respect to the radar (R, E). Angle δ represents the amount of ray bending experienced by the ray passing through the refractive medium. Under these conditions Archer[10] has shown that the angle δ is given by

$$\delta = \pi - 2\left[\gamma_o + \mu(\mathscr{R}_o)\cos^{-1}\left(\frac{\mathscr{R}_o}{R_o}\right)\right] \tag{89}$$

The three plotted rays have actually a small third dimensional component. Because of this, the accuracy of these plotted rays is approximately (+3, -2) percent.

As shown by the tabulated results in Table 1, the computed deviation angle δ_c, arising from ray trace results, agrees very well with the calculated angle, δ, obtained by use of Equation 89. These computer

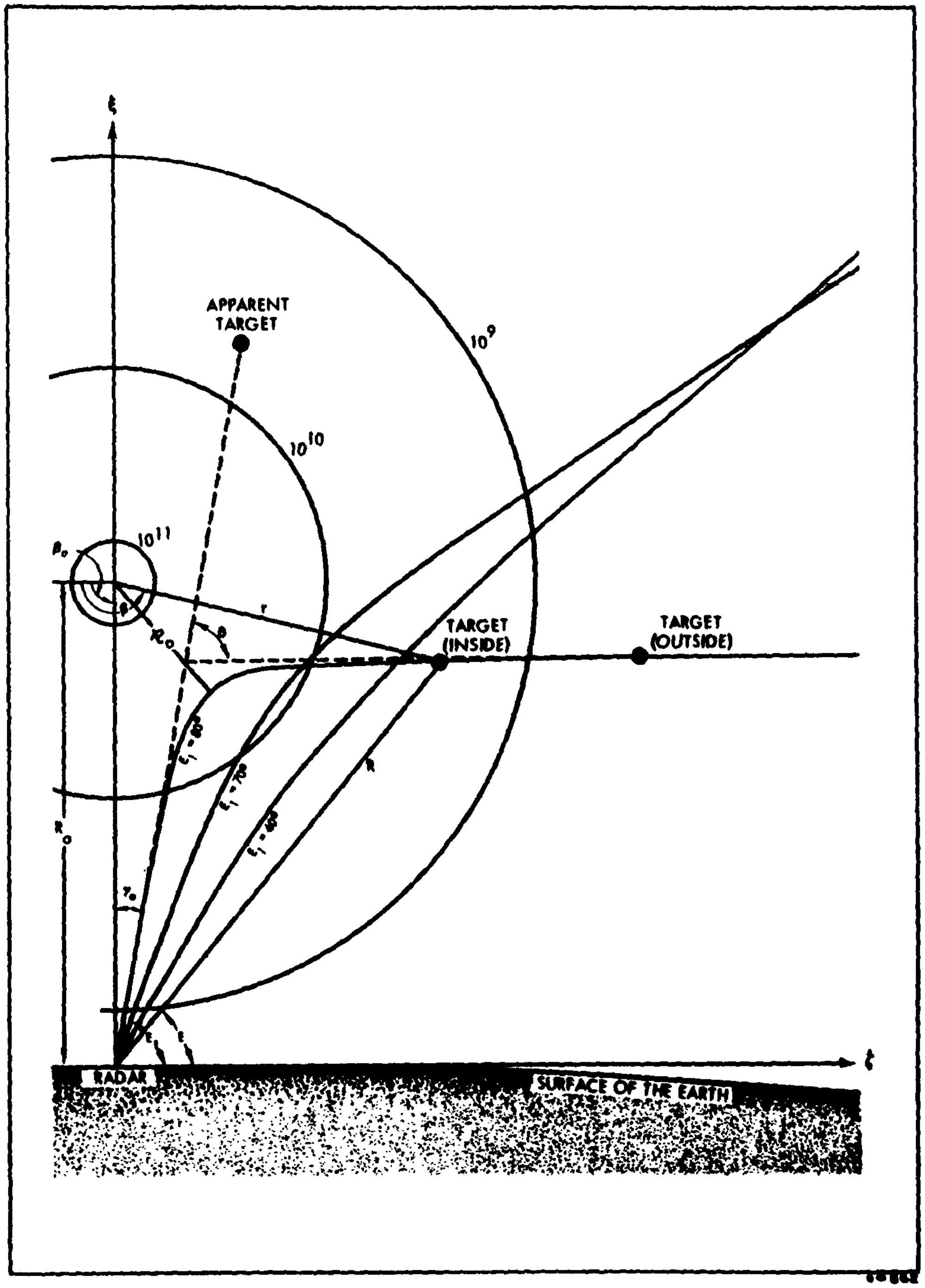


Figure 3. Geometry for Refraction by Spherically Stratified Region

calculations were performed under a large error upper bound condition ($\bar{E}$ = 10^{-3}, see description of Subroutine INT). The agreement is improved by a variation of this error condition. Additional results will not be presented here. Presented elsewhere[12] is the influence of $\bar{E}$ on computed results using these numerical methods as applied to the study of ionization-deionization phenomena.

Radar Elevation Angle - E Degrees	Bending Angle - δ from Equation 89 Degrees	Bending Angle - δ_c Ray Trace Program Degrees
60	19.5	20
70	35.2	37
80	77.5	78

Table 1

Comparison of Total Ray Bending Angle

As an additional illustration, refractive errors through a particular spherical ionized region have been computed in detail to illustrate the concepts discussed and to indicate the kinds of refractive errors that could arise. The electron density contours of this ionization model are defined by Equation 74 where $A = 10^{33}$ and $n = 12$. The distance from the center of the spherical ionosphere is measured in kilometers. The center of the model is located at an elevation angle, E, of 30 degrees, zero degree azimuth angle, and 564 kilometers slant range as measured from the radar site.

Figure 4 shows the relation between the radar and ionization model in the plane of zero azimuth. Also shown are the ray paths for rays leaving the radar at several elevation angles. A frequency of one kilomegacycle was used in determining the refraction of the electromagnetic wave propagation vector. Because the electron density increases rapidly near the center of the model, there is significant bending of the ray path.

The refraction becomes so severe as the elevation angle of radar rays approach the elevation angle of the ionization center with respect to the radar, that there is a region (shown by half tones) into which no radar ray penetrates, hence, radar "blackout" is achieved. In three dimensions this blackout region is a cone in which a target is shielded from the radar. Because rays near this region intersect each other, two elevation angle paths to the same target exist, so multiple targets may be visible.

If the ray path is not in the zero azimuth plane, the amount of elevation error, or azimuth error, is a function of the location of the target. The elevation and azimuth errors for a target located at a slant range of 1200 kilometers have been computed as a function of radar elevation and azimuth angles. These are summarized in Figure 5, in which contours of constant elevation error, ΔE, in one quadrant and constant azimuth error, ΔA, in another quadrant as a function of the ray direction at tne radar site are given. The contours have been terminated at a total bearing error of about 10 degrees. Due to symmetry the errors in the other quadrants are just the mirror image of the quadrants shown.

Nomenclature Used for Computational Results

E_1	initial elevation angle made by ray path
$\mathscr{L}_o$	distance of closest approach between ray path and center of sphere
$(\mathscr{L}, \beta)$	coordinates of any point on the ray path with respect to the center of the sphere
(R, E)	coordinates of any point on the ray path with respect to the radar
R_o	distance between the radar and center of the sphere
γ_o	apparent bearing angle

SECTION III

COMPUTER PROGRAM FOR THREE-DIMENSIONAL RAY-TRACING

Figure 6 schematically describes the computer program that was developed for three-dimensional ray tracing. As illustrated, the computer program is a composite of a group of subprograms. Because each subroutine is an entity in itself, the improvement of the entire program can be performed by the variation of each subprogram.

For the creation of this program the FORTRAN language[13] was used wherever possible. FORTRAN is an automatic coding.system for the IBM-704/709/7090 Data Processing Computer System that was designed for scientific application. Although there are limitations to FORTRAN, nevertheless, 1) it is at present the only language for scientific use, that is accepted by most existing large computer systems, and 2) it is simple and therefore without much effort, permits the elimination of the programmer, thus leaving the design of logical computer decisions, to the formulator of the scientific problem. The program has been written to operate "in or out" of the FORTRAN MONITOR CONTROL SYSTEM.

Except for the RINDEX subroutine the program has been divided into small, simple Functions and Subroutines to facilitate understanding. In the development of the program, concentration was mainly on obtaining a correct working program, as soon as possible, and not on optimization or clarity of output results. These tasks are left for future development.

The computer program consists of the following parts:

1)	Main Program	RAY TRACE
2)	Function	SLANTR
3)	Function	QATAN
4)	Function	ARCOS

5) Subroutine	COORD
6) Subroutine	DAUX
7) Subroutines	INT and INTM
8) Subroutine	RINDEX
9) Subroutine	ELECTX
10) Subroutine	BIGR
11) Subroutine	MAGY
12) Subroutine	COLFRZ
13) Subroutine	RCOORD
14) Subroutine	OUTONE
15) Subroutine	OUTPUT

These functions and subroutines are used to obtain numerical values for those variables which cannot be defined by only one arithmetic statement. In addition to these subprograms certain statements in the FORTRAN language cause the inclusion in the object program of the necessary input and output routines, as well as, various library functions and subroutines in relocatable binary form that are available on the FORTRAN MASTER LIBRARY TAPE. The names and locations of these necessary routines are given in the "storage map" of the arrangement of storage location in the object program that is compiled from a FORTRAN source program. These "maps" follow the listings of each source program. These added routines will not be discussed in this report. Following a brief description of the function of the main program and its associated subprograms, the necessary input data for a sample calculation is given with a description of the output. Some of the results listed in this output led to the graphical results presented in Figures 3, 4 and 5.

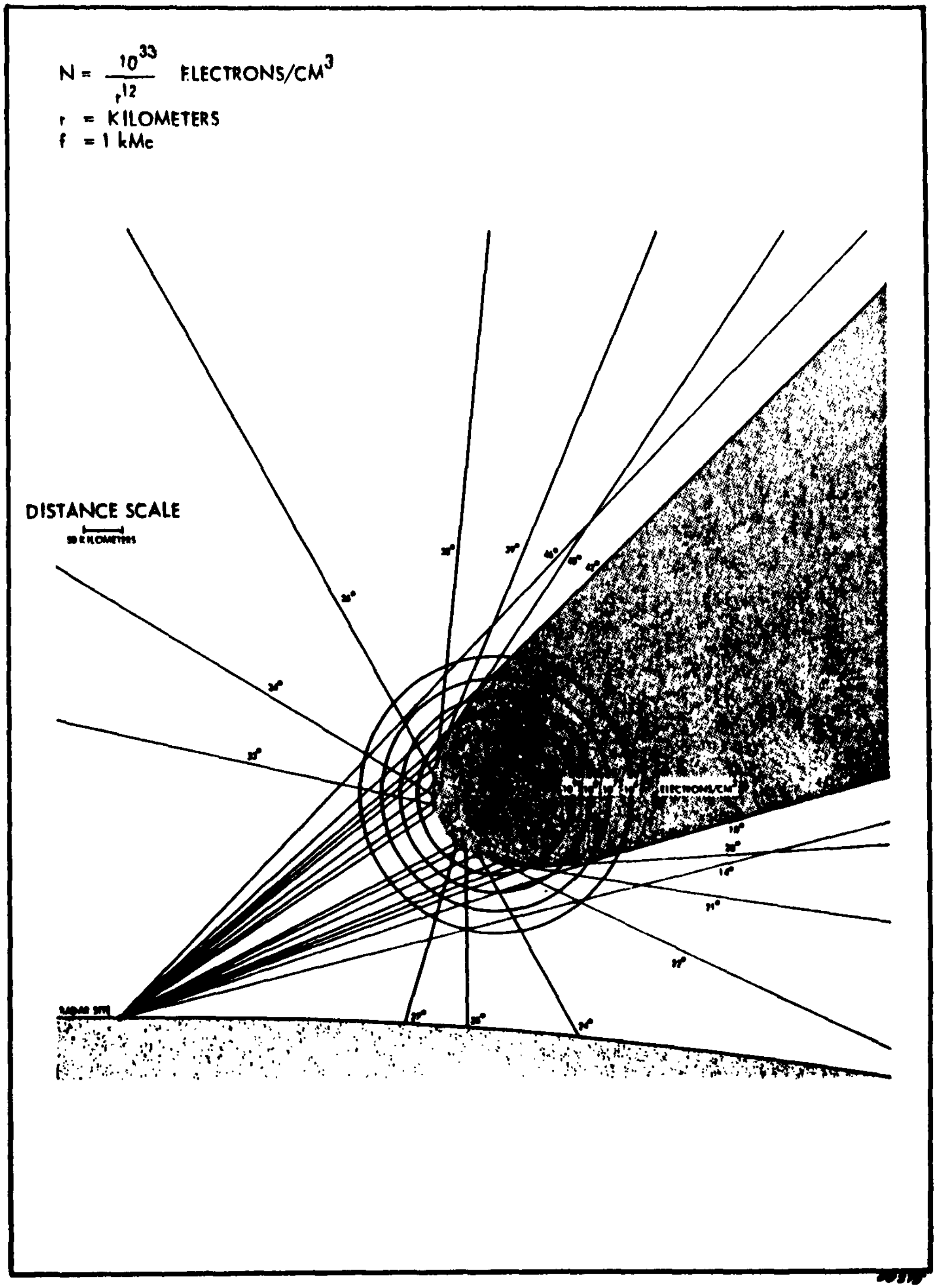


Figure 4. Radar Propagation Paths through Spherically Ionized Region

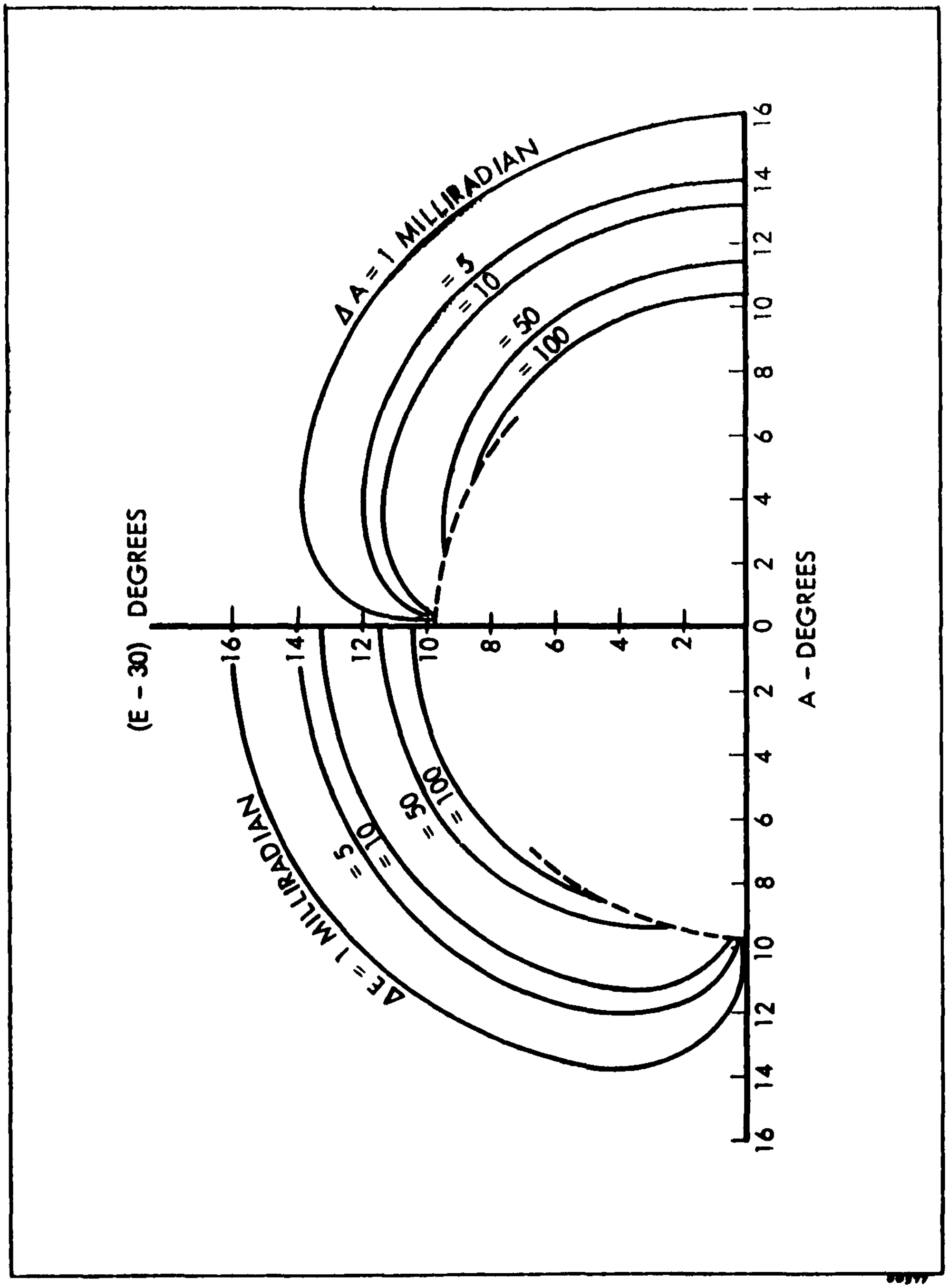


Figure 5. Elevation and Azimuth Errors for Propagation through a Spherical Model - f = 1 kMc

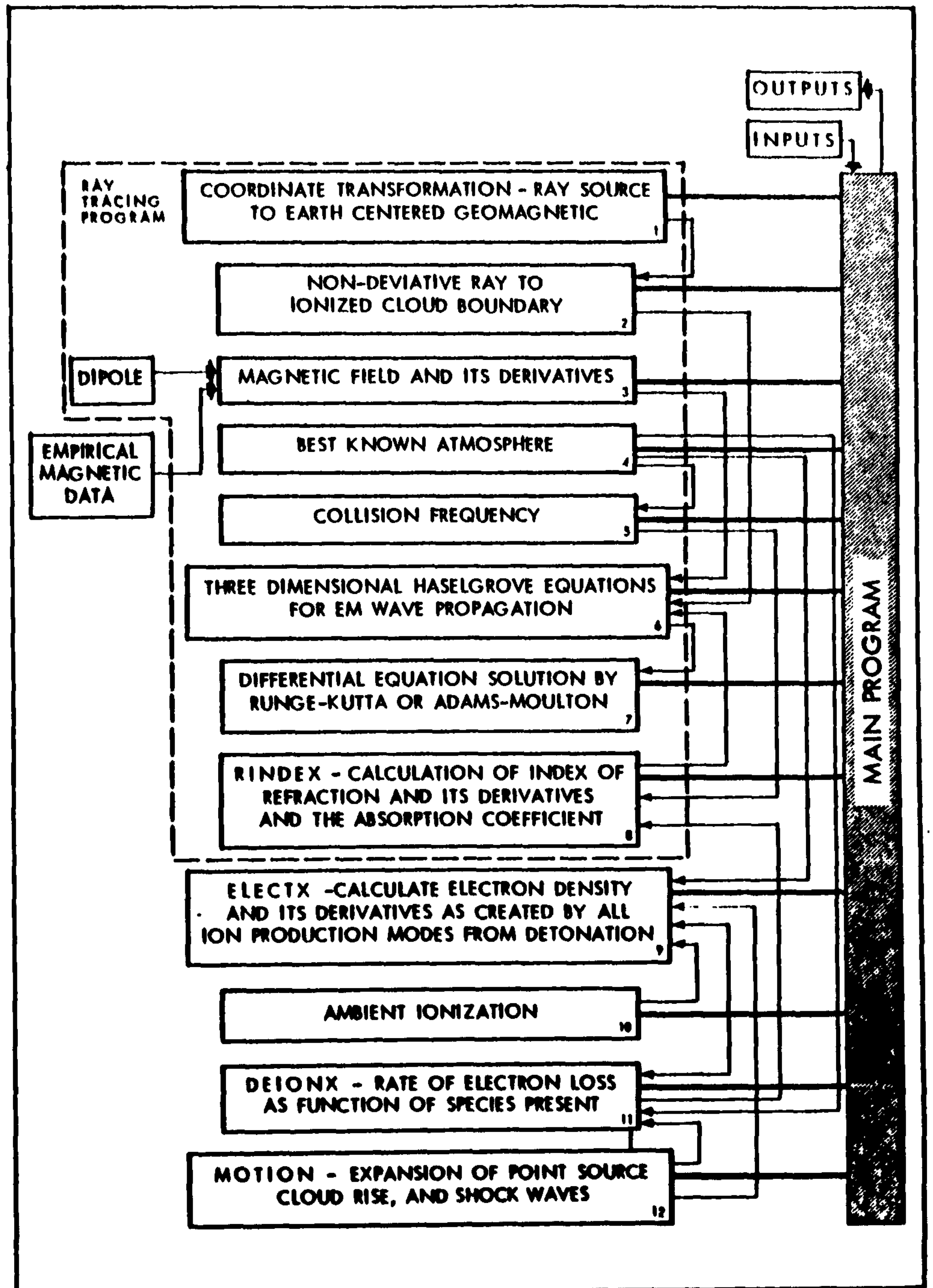


Figure 6. Block Diagram of a Possible Computer Program

A. MAIN PROGRAM RAY TRACE

The main program's function is to act as a master control of the logical flow necessary for the execution of the numerical methods. In addition, it is responsible for obtaining the necessary data, initializing the required starting conditions, performing the desired controlled printouts of computed results, and determining the condition for termination of the given computations. For initializing the starting conditions the main program requires the reading into storage of the following information in the format illustrated under INPUT, Table 4.

CARD 1	RECORD: This can be any desired information consisting of 72 alphanumeric characters that will serve to identify the calculations.
CARD 2	Contains the values of ID, KWIT. ID is an integer that can be used for identifying the calculation if the same CARD 1 is used. KWIT: On completion of the calculations in progress, the computer will check if additional problems are to be performed. Thus if KWIT=59 the computer will want to read a new W vector (CARD 3 --- onward); if KWIT=66, the computer will want to read a new CARD 1, CARD 2, and new W vector (CARD 3 --- onward); if KWIT equals any other integer the computer will PAUSE 44444.
CARD 3 onward	This card and all following cards describe the value of each component of the W vector that is not zero. As shown under INPUT, the first three columns of the card are for the integer that describes the W vector component. The next fourteen columns of the card are for the value of the W vector component. The number of these cards is variable since on completion of one calculation, often only one component of the W vector

CARD 3 (continued) — is to be changed for the next computation. The W vector can be read in any order.

LAST DATA CARD — This card follows the last card describing the W vector. It is any negative integer listed in the first three columns of a card. It transfers the computer out of the read mode to the location beginning the ray trace calculations.

SENSE SWITCH 1 — The program is designed to calculate first the ordinary ray path and then the extraordinary ray path. SENSE SWITCH 1 DOWN will eliminate the calculation of the extraordinary ray path.

SENSE SWITCH 2 — In DOWN position will permit the calculation of the extraordinary ray path and eliminate the calculation of the ordinary ray path.

SENSE SWITCH 3 — Placing this SENSE SWITCH 3 DOWN will terminate the calculation on completion of the ray path calculations in progress.

SENSE SWITCH 4 — Placing SENSE SWITCH 4 DOWN will cause the computer to check if SENSE SWITCH 6 is DOWN. If it is down the computer will terminate calculations immediately.

SENSE SWITCH 6 — It is desirable to follow the course of any calculation on a computer. SENSE SWITCH 6 DOWN will print on-line, the total number of numerical integrations completed up to this point, integration mesh size, length of independent variable τ, height above surface of the earth (km), θ, φ (in degrees), σ_r, σ_θ, σ_φ, μ, κ, distance from the ion source center (km), value of the normalized electron density X.

SENSE LIGHT 2	When SENSE SWITCHES 1 and 2 are DOWN, then both ordinary and extraordinary ray paths are being calculated. When SENSE LIGHT 2 is ON, then the calculation is determining the ordinary ray path. When it is OFF, then the extraordinary ray path is being calculated.
PAUSE 17171	If the computer halts with this octal number in the address field of the STORAGE REGISTER it signifies that SENSE SWITCHES 1 and 2 are in the UP position and the problem is undefined. SENSE SWITCHES 1 or 2, or 1 and 2 are to be placed DOWN depending if only the ordinary, the extraordinary, or both ray paths are to be calculated. Following the definition of the problem, pushing START key will cause calculations to resume.
PAUSE 66666	The computer halts with this octal number in the STORAGE REGISTER just prior to beginning calculations. If the MONITOR system is used it permits the operator to know when it has left the MONITOR system and the SENSE SWITCHES can be changed as needed by the problem.
PAUSE 44444	The computer halts with this octal number in the STORAGE REGISTER on completion of all the necessary calculations specified by the INPUT data. It permits the operator to reset the desired sense switches for the MONITOR system. Pressing START will cause the computer to exit from the program to the MONITOR system.

Table 2 contains the nomenclature that describes some of the components of the V vector and the components of the W vector.

V(2)	independent variable τ
V(3)	initial step size input $\Delta\tau$
V(4)	radius from center of earth r
V(5)	variable angle θ
V(6)	variable angle φ
V(7)	σ_r
V(8)	σ_θ
V(9)	σ_φ
V(10)	optical path length one way s
V(11)	time one way T
V(12)	A absorption
V(13)	$dr/d\tau$
V(14)	$d\theta/d\tau$
V(15)	$d\varphi/d\tau$
V(16)	$d\sigma_r/d\tau$
V(17)	$d\sigma_\theta/d\tau$
V(18)	$d\sigma_\varphi/d\tau$
V(19)	$ds/d\tau$
V(20)	$dT/d\tau$
V(21)	$dA/d\tau$

Table 2. Nomenclature Describing the V and W Vectors.
(Page 1 of 6)

W(1)	refractive index μ
W(2)	imaginary part of complex phase refractive index κ
W(3)	radar transmitter angular frequency ω
W(4)	$\partial\mu/\partial\sigma_r$
W(5)	$\partial\mu/\partial\sigma_\theta$
W(6)	$\partial\mu/\partial\sigma_\varphi$
W(7)	$\partial\mu/\partial r$
W(8)	$\partial\mu/\partial\theta$
W(9)	$\partial\mu/\partial\varphi$
W(10)	$\partial\mu/\partial\psi$
W(11)	$\partial\mu/\partial\omega$
W(12)	unassigned for this program
W(13)	geographic longitudinal angle λ_M of geomagnetic north pole measured east of Greenwich Meridian (degrees)
W(14)	angle λ_R measured as W(13) in degrees
W(15)	angle φ_M geographic latitude of geomagnetic north-pole measured plus from geographic equator north
W(16)	angle φ_R geographic latitude of radar (degrees)
W(17)	radar elevation angle E (degrees)
W(18)	radar azimuth bearing angle angle A (degrees)
W(19)	r_o radius of the earth (km)
W(20)	h_S height of starting point above surface of earth

Table 2. Nomenclature Describing the V and W Vectors.
(Page 2 of 6)

W(21)	angle φ_B of ionization source measured as W(15) (degrees)
W(22)	longitudinal angle λ_B of source measured as W(13)
W(23)	h_B height of ionization source center above earth surface
W(24)	$\Delta\tau$ initial mesh size of variable
W(25)	Y_e normalized equator magnetic field on earth's surface at the geomagnetic equator
W(26)	a constant determining collision frequency
W(27)	b constant in exponent determining collision frequency
W(28)	range (km) = distance from ionization source center to spatial point (r, θ, φ) =
W(29)	cosine of angle makes with the vertical through center of the ionizing source
W(30)	R_b radial distance from earth's center to center of ionizing source
W(31)	x geomagnetic coordinate of source
W(32)	y geomagnetic coordinate of source
W(33)	z geomagnetic coordinate of source
W(34)	A = constant in A/R^n determining electron density
W(35)	n = exponent in A/R^n equation
W(36)	unassigned for this program
W(37)	unassigned for this program
W(38)	plasma angular frequency cycles/sec

Table 2. Nomenclature Describing the V and W Vectors.
(Page 3 of 6)

W(39)	N_e in ion pairs/cc
W(40)	maximum r = V(4) to be considered in this calculation
W(41)	A1 = vector in INTM routine If W(41) = 0 routine will use predictor corrector with variable V'(24) If W(41) = 2 will use Runge-Kutta with fixed W(24) If W(41) = 2 will use predictor-corrector with fixed W(24) If W(41) = 1 or 2 then W(42) through W(47) are ignored but must have some value. If W(41) = 0 they are not ignored
W(42)	A2 = E upper bound on truncation error. See upper bound Equation (10) Appendix A in the INT and INTM subroutine
W(43)	A3 = M is value from which lower bound E is calculated LBE = UBE/M in subroutine INT
W(44)	A4 = A as used in truncation error test EQ (10) in subroutine INT
W(45)	A5 = upper bound on mesh size (If = 0 no upper bound as long as within error range)
W(46)	A6 = lower bound on mesh size (If = 0 lower bound = 0)
W(47)	A7 = β, that is, 0 is less than β less than 1. It is used to decrease or increase mesh size by dividing or multiplying current integration mesh being used
W(48)	smallest attenuation to be considered
W(49)	initial refraction index = W(1)
W(50)	initial absorption kappa κ = W(2)

Table 2. Nomenclature Describing the V and W Vectors.
(Page 4 of 6)

W(51)	initial attenuation = A
W(52)	x_R (km) (Radar coordinate in geomagnetic coordinate system)
W(53)	y_R (km) (Radar coordinate in geomagnetic coordinate system
W(54)	z_R (km) (Radar coordinate in geomagnetic coordinate system
W(55)	$R = \sqrt{(x_R - x)^2 + (y_R - y)^2 + (z_R - z)^2}$ (km)
W(56)	$\Delta R = c\tau - W(55) = (2.99791 \times 10^5)[V(11)] - W(55)$ km
W(57)	new elevation angle E in degrees
W(58)	$\Delta E = W(57) - W(17)$ degrees
W(59)	$2[(W(1))(W(2))] / W(1)^2 - W(2)^2$
W(60)	slant range at r, θ, φ
W(61)	angle A at r, θ, φ
W(62)	elevation angle E at r, θ, φ
W(63)	assigned value to k; if 1 then control is on radius; if 2 then control is on range W(28); if 3 then control is on slant range W(60)
W(64)	value of Z
W(65)	value of Y
W(66)	value of X
W(67)	location of sign which determines the calculation for ordinary or extraordinary ray
W(68)	value of V(4) above which RINDEX is to print R vector

Table 2. Nomenclature Describing the V and W Vectors. (Page 5 of 6)

W(69)	value W(1) below which R vector is printed if W(68) = 0
W(70)	number of performed integrations
W(71)	a in COLFRZ 100-200 km
W(72)	b in COLFRZ 100-200 km
W(73)	a in COLFRZ 200-300 km
W(74)	b in COLFRZ 200-300 km
W(75)	a in COLFRZ 300-400 km
W(76)	b in COLFRZ 300-400 km

W(77) to W(250) unassigned in this program

Table 2. Nomenclature Describing the V and W Vectors.
(Page 6 of 6)

```
C      MAIN  PROGRAM - RAY TRACE  JANUARY 20. 1961    IBM-7090
C     SWITCH 1 DOWN  CALCULATE ORDINARY RAY ONLY
C     SWITCH 2 DOWN  CALCULATE EXTRA-ORDINARY RAY
C     SENSE SWITCH 3 DOWN WILL EXIT AFTER COMPLETING  THIS RAY
C     SENSE SWITCH 4 WITH SENSE SWITCH 6 DOWN WILL EXIT
C     SENSE SWITCH 6 DOWN PERMITS ON-LINE PRINTING OF CALCULATIONS IN
C     PROGRESS INT,V(3),V(2),V(4)-RO,V(5),V(6),V(7),V(8),V(9),W(1),W(2),
C     W(23),W(66)
C     PAUSE 66666  READY TO EXECUTE SET SENSE SWITCHES
C     PAUSE 17171   FORGOT TO SET SS1 AND SS2 FOR PROPER RAY CALCULATION
C     PAUSE 44444 READY TO EXIT SET SENSE SWITCHES FOR MONITOR
      COMMON RECORD,V,W,N,XN,YN,ZN,G
      DIMENSION RECORD(12),V(11),W(250)
      DIMENSION XN(7),YN(7),ZN(8),G(3,3)
      PRINT 111
      PAUSE 66666
      PRINT 114
 66   READ INPUT TAPE 5,67,(RECORD(I),I=1,12)
 67   FORMAT (12A6)
      READ INPUT TAPE 5,113,ID,KWIT
 59   READ INPUT TAPE 5,60,K,DATA
 60    FORMAT (I3,E14.7 )
 61    IF(K) 100,62,62
 62  0IF ((K-13)*(K-14)*(K-15)*(K-16)*(K-17)*(K-18)*(K-21)*(K-22)) 64,
     163,64
 63    DATA = DATA 57.29578
 64    W(K) = DATA
 65    GO TO 59
 100   SENSE LIGHT 0
 101   IF (SENSE SWITCH 1) 102,106
 102   SENSE LIGHT 1
 103   IF(SENSE SWITCH 2) 104,1
 104   SENSE LIGHT 2
 105    GO TO 1
 106   IF(SENSE SWITCH 2) 104,107
 107   PRINT 110
       PAUSE 17171
 108   GO TO 100
 110   FORMAT (46HO SET SS1 AND SS2 FOR PROBLEM BEING CONSIDERED)
 111   FORMAT(54HO SET ALL SENSE SWITCHES IN     POSITION DURING EXECUTE)
 112   FORMAT(48HO SET SENSE SWITCHES FOR MONITOR - READY TO EXIT)
 113   FORMAT(2I6)
 114  0FORMAT (5HO INT,4HDELV,5X,4HV(2),4X,7HV(4)-RO,3X,5HTHETA,5X,3HPHI,
      15X,4HSIGR,3X,8HSIGTHETA,6X,6HSIGPHI,5X,4HW(1),6X,4HW(2),5X,5HRANGE
      2,7X,1HX)
 115   FORMAT(1HO,I5,1P7E9.2,3H    ,1PE9.2,1H ,1PE9.2,2H  ,1P3E9.2)
 1     ARG1R = SINF( W(13)-W(14))
       ARG2R = COSF (W(13)-W(14))
       ARG3M = SINF(W(15))
       ARG4M = COSF(W(15))
       ARG5R = SINF(W(16))
       ARG6R = COSF(W(16))
       ARG7R = SINF(W(17))
       ARG8R = COSF(W(17))
       ARG9R = SINF(W(18))
       ARG10R= COSF(W(18))
       IM = W(63)
       PI= 3.1415927
```

```
      G(1,1)= ARG2R*ARG3M*ARG6R - ARG4M*ARG5R
      G(1,2) = ARG1R*ARG3M
      G(1,3) = - ARG5R*ARG3M*ARG2R - ARG6R*ARG4M
      G(2,1) = - ARG1R*ARG6R
      G(2,2) = ARG2R
      G(2,3) = ARG5R*ARG1R
      G(3,1) = ARG6R*ARG4M*ARG2R - ARG3M*ARG5R
      G(3,2) = ARG4M*ARG1R
      G(3,3) = - ARG5R*ARG4M*ARG2R + ARG3M*ARG6R
      XN(1)  = G(2,2)*G(3,3)
      XN(2)  = G(1,2)*G(2,3)
      XN(3)  = G(1,3)*G(3,2)
      XN(4)  =  G(1,3)*G(2,2)
      XN(5)  = G(1,2)*G(3,3)
      XN(6)  = G(3,2)*G(2,3)
      XN(7)  = W(19)*G(1,1)
      YN(1)  = G(1,1)*G(3,3)
      YN(2)  = G(2,3)*G(3,1)
      YN(3)  = G(1,3)*G(2,1)
      YN(4)  = G(1,3)*G(3,1)
      YN(5)  = G(2,1)*G(3,3)
      YN(6)  = G(1,1)*G(2,3)
      YN(7)  = W(19)*G(2,1)
      ZN(1)  = G(1,1)*G(2,2)
      ZN(2)  = G(1,2)*G(3,1)
      ZN(3) = G(2,1)*G(3,2)
      ZN(4)  = G(2,2)*G(3,1)
      ZN(5)  = G(1,2)*G(2,1)
      ZN(6)  = G(1,1)*G(3,2)
      ZN(7)  = W(19)*G(3,1)
      DENM = ZN(1)*G(3,3) + ZN(2)*G(2,3) + ZN(3)*G(1,3) - G(1,3)*ZN(4)
     1  - ZN(5)*G(3,3) - G(2,3)*ZN(6)
      ZN(8) = DENM
2     SRSP = SLANTR (ARG7R,ARG9R,W(19),W(20))
       N = 9
3    OCALL COORD (XSP,YSP,ZSP,ARG1R,ARG2R,ARG3M,ARG4M,ARG5R,ARG6R,ARG7R,
     1 ARG8R,ARG9R,ARG10R,SRSP,W(19))
4     RSP = SQRTF (XSP**2+YSP**2+ZSP**2)
5     QSP=  ZSP/RSP
6     THETSP = ARCOS(QSP)
7     PHISP  =  QATAN(XSP,YSP)
8     ZERO   = 0.0
9    OCALL  COORD (XR,YR,ZR,ARG1R,ARG2R,ARG3M,ARG4M, ARG5R,ARG6R,ARG7R,
     1ARG8R,ARG9R,ARG10R,ZERO,W(19))
10    QR= ZR/W(19)
      W(52) = XR
      W(53) = YR
      W(54) = ZR
11    THETR = ARCOS(QR)
12    PHIR   =  QATAN(XR,YR)
13    ARGSE =  (((W(19))/RSP)*ARG8R)
14    TERM2 =(QSP*QSP)
15    ARG1B = SINF (W(13)  - W(22))
      ARG2B=COSF(W(13)-W(22))
      ARG5B=SINF(W(21))
      ARG6B=COSF(W(21))
16   OCALL COORD (XO,YO,ZO,ARG1B,ARG2B,ARG3M,ARG4M,ARG5B,ARG6B,ZERO,
     1ZERO,ZERO,ZERO,ZERO,W(19))
```

```
17    W(30)=W(19)+W(23)
      QB =  ZO/W(19)
      THETB = ARCOS(QB)
       PHIB =  QATAN (XO,YO )
18    W(33)= (ZO/W(19))*W(30)
19    W(31)= (XO/W(19))*W(30)
20    W(32)= (YO/W(19))*W(30)
22    TERM4 =  (PHISP-PHIR)
23    TERM5 = SQRTF( 1.0- QR*QR)
24   0TERM6 =  TERM5* SINF(TERM4)
25    TERM7 =  QSP*TERM5*COSF(TERM4)- (SQRTF(1.0 -TERM2))*QR
26    ANALPH =  QATAN(TERM7,TERM6)
28    SIGR  =W(1) * SQRTF(1.0-ARGSE**2)
      PI = 3.1415927
        GAMMA  = ANALPH - PI
29    SIGTHE =(W(1)* ARGSE)* COSF(GAMMA)
30    SIGPHI= -((W(1)*ARGSE)*SINF(GAMMA))
      NA = W(41)
31    IF (SENSE LIGHT 1) 32, 34
32    SIGN = 1.0
33    GO TO 36
34    IF(SENSE LIGHT 2) 35,200
35    SIGN =  -1.0
36    IF(SENSE SWITCH 3) 37,39
37    PRINT 112
      PAUSE 44444
      CALL EXIT
38    W(1)= W(49)
      W(67) = SIGN
       W(70) = 0.0
39    V(2)= SRSP
40    V(3)= W(24)
41    V(4)= RSP
42    V(5)= THETSP
43    V(6)= PHISP
44    V(7)= SIGR
45    V(8)= SIGTHE
46    V(9) = SIGPHI
47    V(10) = SRSP
48    V(11) =(SRSP/3.0E 5)
49    V(12) = W(51)
50    W(2) = W(50)
      ID = ID + 1
      CALL OUTONE(SIGN,ID)
53    CALL  INT (V, N, NA  ,W(42),W(43),W(44),W(45),W(46),W(47))
51    IF(V(4) - W(19))31,31,54
54    CALL  INTM
      IF (SENSE SWITCH 6)116,117
117   IF(W(1))118,118,56
118   PRINT 119
119   FORMAT(28H0  W(1)  IS NEGATIVE OR ZERO)
116   TE3 = V(4)- W(19)
      TE1 = V(5)*57.29578
      TE2 = V(6)*57.29578
      INT = W(70)
      PRINT 115,INT,V(3),V(2),TE3,TE1,TE2,V(7),V(8),V(9),W(1),W(2),W(28)
     1,W(68)
55     CALL OUTPUT
```

```
      IF (SENSE SWITCH 4)37,57
57    GO TO (202,203,204),IM
202   IF(V(4) - W(40))51,51,31
203   IF(W(28)- W(40))54,54,31
204   IF(W(60)- W(40))54,54,31
200   IF(KWIT-59)201,59,201
201   IF(KWIT-66) 37,66,37
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
G	32165	76645	N	32188	76674	RECORD	32561	77461	V	32549	77445	W	32438	77266
XN	32187	76673	YN	32180	76664	ZN	32173	76655						

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
67	4	00000	60	5	00000	110	6	00000	111	7	00000	112	8	00000
113	9	00000	114	10	00000	115	11	00000	119	12	00000	66	17	00035
66	21	00047	59	24	00066	59	25	00073	61	26	00103	62	27	00107
63	28	00170	64	29	00173	65	30	00175	100	31	00177	101	32	00200
102	33	00203	103	34	00204	104	35	00206	105	36	00207	106	37	00210
107	38	00214	107	38	00214	108	40	00220	1	41	00221	2	85	00510
2	86	00510	3	88	00520	3	89	00520	4	90	00540	5	91	00554
6	92	00557	6	93	00557	7	94	00562	7	95	00562	8	96	00566
9	97	00570	9	98	00570	10	99	00610	11	103	00621	11	104	00621
12	105	00624	12	106	00624	13	107	00630	14	108	00634	15	109	00637
16	113	00655	16	114	00655	17	115	00675	18	121	00712	19	122	00716
20	123	00722	22	124	00726	23	125	00731	24	126	00737	25	127	00745
26	128	00763	26	129	00763	28	130	00767	29	133	01005	30	134	01015
31	136	01035	32	137	01037	33	138	01041	34	139	01042	35	140	01045
36	141	01047	37	142	01054	37	142	01054	38	145	01061	39	148	01067
40	149	01071	41	150	01073	42	151	01075	43	152	01077	44	153	01101
45	154	01103	46	155	01105	47	156	01107	48	157	01111	49	158	01114
50	159	01116	53	163	01126	53	164	01126	51	165	01140	54	166	01145
117	168	01152	118	169	01155	118	169	01155	116	170	01160	56	176	01235
57	178	01242	202	179	01245	203	180	01252	204	181	01257	200	182	01265
201	183	01271												

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
867	01543	32156	76634

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
INT	14	00016	INTM	15	00017	EXIT	12	00014	COS	6	00006	COORD	8	00010
ARCOS	10	00012	OUTONE	13	00015	OUTPUT	16	00020	QATAN	11	00013	SIN	5	00005
SLANTR	7	00007	SQRT	9	00011	(FIL)	2	00002	(FPT)	0	00000	(RTN)	4	00004
(SPH)	1	00001	(TSH)	3	00003									

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
IM	866	01542	ID	865	01541	GAMMA	864	01540	DENM	863	01537	DATA	862	01536
ARGSE	861	01535	ARG9R	860	01534	ARG8R	859	01533	ARG7R	858	01532	ARG6R	857	01531
ARG6B	856	01530	ARG5R	855	01527	ARG5B	854	01526	ARG4M	853	01525	ARG3M	852	01524
ARG2R	851	01523	ARG2B	850	01522	ARG1R	849	01521	ARG1B	848	01520	ARG10R	847	01517
ANALPH	846	01516	K	845	01515	KWIT	844	01514	NA	843	01513	PHIB	842	01512
PHIR	841	01511	PHISP	840	01510	PI	839	01507	QB	838	01506	QR	837	01505
QSP	836	01504	RSP	835	01503	SIGN	834	01502	SIGPHI	833	01501	SIGR	832	01500
SIGTHE	831	01477	SPSP	830	01476	TE1	829	01475	TE2	828	01474	TE3	827	01473
TERM2	826	01472	TERM4	825	01471	TERM5	824	01470	TERM6	823	01467	TERM7	822	01466
THETB	821	01465	THETR	820	01464	THETSP	819	01463	XO	818	01462	XR	817	01461

```
      XSP   816 01460        YO   815 01457        YR   814 01456        YSP   813 01455        ZO   812 01454
     ZERO   811 01453        ZR   810 01452       ZSP   809 01451

                     STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

           DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
    E)20J   553 01051     E)109      126 00176     E)10      696 01270     E)R       673 01241     E)N       617 01151
    E)H     548 01044     E)C        139 00213     E)9       130 00202     E)6       119 00167     D)601      27 00033
    D)40V   692 01264     D)401       28 00034     D)10J     555 01053     C)G3      808 01450     C)G1      807 01447
    8)3N    731 01333     8)3J       740 01344     8)3I      763 01373     8)3H      764 01374     8)3G      773 01405
    8)3F    783 01417     8)3E       792 01430     8)23      797 01435     8)1S      795 01433     1)        798 01436
    2)      702 01276     3)         716 01314     4)      32767 77777     6)        721 01321

                        ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
     (FPT)       (SPH)       (FIL)       (TSH)       (RTN)        SIN        COS       SLANTR       COORD       SQRT
     ARCOS       QATAN        EXIT       OUTONE        INT        INTM      OUTPUT
```

```
00000 (FPT)  BCD 1(FPT)
00001 (SPH)  BCD 1(SPH)
00002 (FIL)  BCD 1(FIL)
00003 (TSH)  BCD 1(TSH)
00004 (RTN)  BCD 1(RTN)
00005 SIN    BCD 1SIN
00006 COS    BCD 1COS
00007 SLANTR BCD 1SLANTR
00010 COORD  BCD 1COORD
00011 SQRT   BCD 1SQRT
00012 ARCOS  BCD 1ARCOS
00013 QATAN  BCD 1QATAN
00014 EXIT   BCD 1EXIT
00015 OUTONE BCD 1OUTONE
00016 INT    BCD 1INT
00017 INTM   BCD 1INTM
00020 OUTPUT BCD 1OUTPUT
00021 $$     CLA (FPT)
00022        STO 8
00023        STZ 4)-205
       14A   BSS
00024        TSX (SPH),4
00025        PZE 8)3F
             BSS
00026        TSX (FIL),4
00027 15A    HPR 28086
       16A   BSS
00030        TSX (SPH),4
00031        PZE 8)3I
             BSS
00032        TSX (FIL),4
00033 D)601  LXD C)G3,2
00034 D)401  LXD C)G1,4
00035 17A    CAL 2)
00036        SXD 6)+4,4
00037        TSX (TSH),4
00040        PZE 8)23
00041        LXD 6)+4,4
00042 18A    LXD 2)+11,1
00043 19A    STR
00044        STQ RECORD+1,1
00045 19A1   TXI *+1,1,1
00046 19A2   TXL 19A,1,12
00047 21A    SXD 6)+4,4
00050        TSX (RTN),4
00051        LXD 6)+4,4
00052 22A    CAL 2)
00053        SXD 6)+4,4
00054        TSX (TSH),4
00055        PZE 8)3H
00056        LXD 6)+4,4
00057 23A    STR
00060        STQ ID
00061        STR
00062        STQ KWIT
00063        SXD 6)+4,4
00064        TSX (RTN),4
00065        LXD 6)+4,4
```

```
00066 24A    CAL 2)
00067        SXD 6)+4,4
00070        TSX (TSH),4
00071        PZE 8)15
00072        LXD 6)+4,4
00073 25A    STR
00074        STQ K
00075        STR
00076        STQ DATA
00077        SXD 6)+4,4
00100        TSX (RTN),4
00101        LXD 6)+4,4
00102        LXD K,1
00103 26A    CLA K
00104 26A1   TZE 27A
00105        TPL 27A
00106        TRA E)108
00107 27A    CLA K
00110        SUB 2)+8
00111        STO 1)+1
00112        CLA K
00113        SUB 2)+7
00114        STO 1)+2
00115        CLA K
00116        SUB 2)+6
00117        STO 1)+3
00120        CLA K
00121        SUB 2)+5
00122        STO 1)+4
00123        CLA K
00124        SUB 2)+4
00125        STO 1)+5
00126        CLA K
00127        SUB 2)+3
00130        STO 1)+6
00131        CLA K
00132        SUB 2)+2
00133        STO 1)+7
00134        CLA K
00135        SUB 2)+1
00136        STO 1)+8
00137        LDQ 1)+1
00140        MPY 1)+8
00141        ALS 17
00142        XCA
00143        MPY 1)+7
00144        ALS 17
00145        XCA
00146        MPY 1)+6
00147        ALS 17
00150        XCA
00151        MPY 1)+5
00152        ALS 17
00153        XCA
00154        MPY 1)+4
00155        ALS 17
00156        XCA
00157        MPY 1)+3
```

```
00160        ALS 17
00161        XCA
00162        MPY 1)+2
00163        ALS 17
00164 27A1   TZE E)6
00165        TPL 29A
00166        TRA 29A
00167 E)6    SXD C)G1,1
00170 28A    CLA DATA
00171        FDP 3)
00172        STQ DATA
00173 29A    CLA DATA
00174        STO W+1,1
00175 30A    TRA 24A
00176 E)108  SXD C)G1,1
00177 31A    PSE 96
00200 32A    PSE 113
00201        TRA 37A
00202 E)9    SXD C)G3,2
00203 33A    PSE 97
00204 34A    PSE 114
00205        TRA 41A
00206 35A    PSE 98
00207 36A    TRA 41A
00210 37A    PSE 114
00211        TRA E)C
00212        TRA 35A
00213 E)C    SXD C)G3,2
       38A   BSS
00214        TSX (SPH),4
00215        PZE 8)3E
             BSS
00216        TSX (FIL),4
00217 39A    HPR 7801
00220 40A    TRA 31A
00221 41A    CLA W-12
00222        FSB W-13
             BSS
00223        TSX SIN,4
00224        STO ARG1R
00225 42A    CLA W-12
00226        FSB W-13
             BSS
00227        TSX COS,4
00230        STO ARG2R
00231 43A    CLA W-14
             BSS
00232        TSX SIN,4
00233        STO ARG3M
00234 44A    CLA W-14
             BSS
00235        TSX COS,4
00236        STO ARG4M
00237 45A    CLA W-15
             BSS
00240        TSX SIN,4
00241        STO ARG5R
00242 46A    CLA W-15
```

```
               BSS
00243          TSX COS,4
00244          STO ARG6R
00245 47A      CLA W-16
               BSS
00246          TSX SIN,4
00247          STO ARG7R
00250 48A      CLA W-16
               BSS
00251          TSX COS,4
00252          STO ARG8R
00253 49A      CLA W-17
               BSS
00254          TSX SIN,4
00255          STO ARG9R
00256 50A      CLA W-17
               BSS
00257          TSX COS,4
00260          STO ARG10R
00261 51A      CLA W-62
00262          UFA 6)
00263          LRS
00264          ANA 6)+1
00265          LLS
00266          ALS 18
00267          STO IM
00270          LXD IM,2
00271 52A      CLA 3)+1
00272          STO PI
00273 53A      LDQ ARG4M
00274          FMP ARG5R
00275          STO 1)+1
00276          LDQ ARG6R
00277          FMP ARG2R
00300          XCA
00301          FMP ARG3M
00302          FSB 1)+1
00303          STO G
00304 54A      LDQ ARG1R
00305          FMP ARG3M
00306          STO G-3
00307 55A      LDQ ARG6R
00310          FMP ARG4M
00311          STO 1)+1
00312          LDQ ARG2R
00313          FMP ARG5R
00314          XCA
00315          FMP ARG3M
00316          CHS
00317          FSB 1)+1
00320          STO G-6
00321 56A      LDQ ARG1R
00322          FMP ARG6R
00323          CHS
00324          STO G-1
00325 57A      CLA ARG2R
00326          STO G-4
00327 58A      LDQ ARG5R
00330          FMP ARG1R
00331          STO G-7
00332 59A      LDQ ARG3M
00333          FMP ARG5R
00334          STO 1)+1
00335          LDQ ARG2R
00336          FMP ARG6R
00337          XCA
00340          FMP ARG4M
00341          FAD 1)+1
00342          STO G-2
00343 60A      LDQ ARG4M
00344          FMP ARG1R
00345          STO G-5
00346 61A      LDQ ARG3M
00347          FMP ARG6R
00350          STO 1)+1
00351          LDQ ARG2R
00352          FMP ARG5R
00353          XCA
00354          FMP ARG4M
00355          CHS
00356          FAD 1)+1
00357          STO G-8
00360 62A      LDQ G-4
00361          FMP G-8
00362          STO XN
00363 63A      LDQ G-3
00364          FMP G-7
00365          STO XN-1
00366 64A      LDQ G-6
00367          FMP G-5
00370          STO XN-2
00371 65A      LDQ G-6
00372          FMP G-4
00373          STO XN-3
00374 66A      LDQ G-3
00375          FMP G-8
00376          STO XN-4
00377 67A      LDQ G-5
00400          FMP G-7
00401          STO XN-5
00402 68A      LDQ W-18
00403          FMP G
00404          STO XN-6
00405 69A      LDQ G
00406          FMP G-8
00407          STO YN
00410 70A      LDQ G-7
00411          FMP G-2
00412          STO YN-1
00413 71A      LDQ G-6
00414          FMP G-1
00415          STO YN-2
00416 72A      LDQ G-6
00417          FMP G-2
00420          STO YN-3
00421 73A      LDQ G-1
00422          FMP G-8
00423          STO YN-4
00424 74A      LDQ G
00425          FMP G-7
00426          STO YN-5
00427 75A      LDQ W-18
00430          FMP G-1
00431          STO YN-6
00432 76A      LDQ G
00433          FMP G-4
00434          STO ZN
00435 77A      LDQ G-3
00436          FMP G-2
00437          STO ZN-1
00440 78A      LDQ G-1
00441          FMP G-5
00442          STO ZN-2
00443 79A      LDQ G-4
00444          FMP G-2
00445          STO ZN-3
00446 80A      LDQ G-3
00447          FMP G-1
00450          STO ZN-4
00451 81A      LDQ G
00452          FMP G-5
00453          STO ZN-5
00454 82A      LDQ W-18
00455          FMP G-2
00456          STO ZN-6
00457 83A      LDQ G-7
00460          FMP ZN-5
00461          STO 1)+1
00462          LDQ ZN-4
00463          FMP G-8
00464          STO 1)+2
00465          LDQ G-6
00466          FMP ZN-3
00467          STO 1)+3
00470          LDQ ZN-2
00471          FMP G-6
00472          STO 1)+4
00473          LDQ ZN-1
00474          FMP G-7
00475          STO 1)+5
00476          LDQ ZN
00477          FMP G-8
00500          FAD 1)+5
00501          FAD 1)+4
00502          FSB 1)+3
00503          FSB 1)+2
00504          FSB 1)+1
00505          STO DENM
00506 84A      CLA DENM
00507          STO ZN-7
      85A      BSS
00510 86A      TSX SLANTR,4
00511          TSX ARG7R
00512          TSX ARG8R
```

```
00513        TSX W-18
00514        TSX W-19
00515        STO SRSP
00516 87A    CLA 2)+9
00517        STO N
      88A    BSS
00520 89A    TSX COORD,4
00521        TSX XSP
00522        TSX YSP
00523        TSX ZSP
00524        TSX ARG1R
00525        TSX ARG2R
00526        TSX ARG3M
00527        TSX ARG4M
00530        TSX ARG5R
00531        TSX ARG6R
00532        TSX ARG7R
00533        TSX ARG8R
00534        TSX ARG9R
00535        TSX ARG10R
00536        TSX SRSP
00537        TSX W-18
00540 90A    LDQ ZSP
00541        FMP ZSP
00542        STO 1)+1
00543        LDQ YSP
00544        FMP YSP
00545        STO 1)+2
00546        LDQ XSP
00547        FMP XSP
00550        FAD 1)+2
00551        FAD 1)+1
             BSS
00552        TSX SQRT,4
00553        STO RSP
00554 91A    CLA ZSP
00555        FDP RSP
00556        STQ QSP
      92A    BSS
00557 93A    TSX ARCOS,4
00560        TSX QSP
00561        STO THETSP
      94A    BSS
00562 95A    TSX QATAN,4
00563        TSX XSP
00564        TSX YSP
00565        STQ PHISP
00566 96A    CLA 3)+2
00567        STO ZERO
      97A    BSS
00570 98A    TSX COORD,4
00571        TSX XR
00572        TSX YR
00573        TSX ZR
00574        TSX ARG1R
00575        TSX ARG2R
00576        TSX ARG3M
00577        TSX ARG4M

00600        TSX ARG5R
00601        TSX ARG6R
00602        TSX ARG7R
00603        TSX ARG8R
00604        TSX ARG9R
00605        TSX ARG10R
00606        TSX ZERO
00607        TSX W-18
00610 99A    CLA ZR
00611        FDP W-18
00612        STQ QR
00613 100A   CLA XR
00614        STO W-51
00615 101A   CLA YR
00616        STO W-52
00617 102A   CLA ZR
00620        STO W-53
      103A   BSS
00621 104A   TSX ARCOS,4
00622        TSX QR
00623        STO THETR
      105A   BSS
00624 106A   TSX QATAN,4
00625        TSX XR
00626        TSX YR
00627        STO PHIR
00630 107A   CLA W-18
00631        FDP RSP
00632        FMP ARG8R
00633        STO ARGSE
00634 108A   LDQ QSP
00635        FMP QSP
00636        STO TERM2
00637 109A   CLA W-12
00640        FSB W-21
             BSS
00641        TSX SIN,4
00642        STO ARG1B
00643 110A   CLA W-12
00644        FSB W-21
             BSS
00645        TSX COS,4
00646        STO ARG2B
00647 111A   CLA W-20
             BSS
00650        TSX SIN,4
00651        STO ARG5B
00652 112A   CLA W-20
             BSS
00653        TSX COS,4
00654        STO ARG6B
      113A   BSS
00655 114A   TSX COORD,4
00656        TSX XO
00657        TSX YO
00660        TSX ZO
00661        TSX ARG1B
00662        TSX ARG2B

00663        TSX ARG3M
00664        TSX ARG4M
00665        TSX ARG5B
00666        TSX ARG6B
00667        TSX ZERO
00670        TSX ZERO
00671        TSX ZERO
00672        TSX ZERO
00673        TSX ZERO
00674        TSX W-18
00675 115A   CLA W-18
00676        FAD W-22
00677        STO W-29
00700 116A   CLA ZO
00701        FDP W-18
00702        STQ QB
      117A   BSS
00703 118A   TSX ARCOS,4
00704        TSX QB
00705        STO THETB
      119A   BSS
00706 120A   TSX QATAN,4
00707        TSX XO
00710        TSX YO
00711        STO PHIB
00712 121A   CLA ZO
00713        FDP W-18
00714        FMP W-29
00715        STO W-32
00716 122A   CLA XO
00717        FDP W-18
00720        FMP W-29
00721        STO W-30
00722 123A   CLA YO
00723        FDP W-18
00724        FMP W-29
00725        STO W-31
00726 124A   CLA PHISP
00727        FSB PHIR
00730        STO TERM4
00731 125A   LDQ QR
00732        FMP QR
00733        CHS
00734        FAD 3)+3
             BSS
00735        TSX SQRT,4
00736        STO TERM5
00737 126A   CLA TERM4
             BSS
00740        TSX SIN,4
00741        STO 1)+1
00742        LDQ TERM5
00743        FMP 1)+1
00744        STO TERM6
00745 127A   CLA 3)+3
00746        FSB TERM2
             BSS
00747        TSX SQRT,4
```

```
00750          XCA
00751          FMP QR
00752          STO 1)+2
00753          CLA TERM4
               BSS
00754          TSX COS,4
00755          XCA
00756          FMP QSP
00757          XCA
00760          FMP TERM5
00761          FSB 1)+2
00762          STO TERM7
          128A BSS
00763 129A     TSX QATAN,4
00764          TSX TERM7
00765          TSX TERM6
00766          STO ANALPH
00767 130A     LDQ ARGSE
00770          FMP ARGSE
00771          CHS
00772          FAD 3)+3
               BSS
00773          TSX SQRT,4
00774          STO 1)+1
00775          LDQ W
00776          FMP 1)+1
00777          STO SIGR
01000 131A     CLA 3)+1
01001          STO PI
01002 132A     CLA ANALPH
01003          FSB PI
01004          STO GAMMA
01005 133A     CLA GAMMA
               BSS
01006          TSX COS,4
01007          STO 1)+1
01010          LDQ W
01011          FMP ARGSE
01012          XCA
01013          FMP 1)+1
01014          STO SIGTHE
01015 134A     CLA GAMMA
               BSS
01016          TSX SIN,4
01017          STO 1)+1
01020          LDQ W
01021          FMP ARGSE
01022          XCA
01023          FMP 1)+1
01024          CHS
01025          STO SIGPHI
01026 135A     CLA W-40
01027          UFA 6)
01030          LRS
01031          ANA 6)+1
01032          LLS
01033          ALS 18
01034          STO NA

01035 136A     MSE 97
01036          TRA 139A
01037 137A     CLA 3)+3
01040          STO SIGN
01041 138A     TRA 141A
01042 139A     MSE 98
01043          TRA D)40V
01044 E)H      SXD C)G3,2
01045 140A     CLS 3)+3
01046          STO SIGN
01047 141A     PSE 115
01050          TRA 145A
01051 E)20J    SXD C)G3,2
01052          TRA 142A
01053 D)10J    LXD C)G1,1
          142A BSS
01054          TSX (SPH),4
01055          PZE 8)3G
               BSS
01056          TSX (FIL),4
01057 143A     HPR 18724
          144A BSS
01060          TSX EXIT,4
01061 145A     CLA W-48
01062          STO W
01063 146A     CLA SIGN
01064          STO W-66
01065 147A     CLA 3)+2
01066          STO W-69
01067 148A     CLA SRSP
01070          STO V-1
01071 149A     CLA W-23
01072          STO V-2
01073 150A     CLA RSP
01074          STO V-3
01075 151A     CLA THETSP
01076          STO V-4
01077 152A     CLA PHISP
01100          STO V-5
01101 153A     CLA SIGR
01102          STO V-6
01103 154A     CLA SIGTHE
01104          STO V-7
01105 155A     CLA SIGPHI
01106          STO V-8
01107 156A     CLA SRSP
01110          STO V-9
01111 157A     CLA SRSP
01112          FDP 3)+4
01113          STQ V-10
01114 158A     CLA W-50
01115          STO V-11
01116 159A     CLA W-49
01117          STO W-1
01120 160A     CLA ID
01121          ADD 2)+11
01122          STO ID
          161A BSS

01123 162A     TSX OUTONE,4
01124          TSX SIGN
01125          TSX ID
          163A BSS
01126 164A     TSX INT,4
01127          TSX V
01130          TSX N
01131          TSX NA
01132          TSX W-41
01133          TSX W-42
01134          TSX W-43
01135          TSX W-44
01136          TSX W-45
01137          TSX W-46
01140 165A     CLA V-3
01141          FSB W-18
01142 165A1    TZE 136A
01143          TPL 166A
01144          TRA 136A
          166A BSS
01145          TSX INTM,4
01146 167A     PSE 118
01147          TRA E)N
01150          TRA 170A
01151 E)N      SXD C)G3,2
01152 168A     CLA W
01153 168A1    TZE 169A
01154          TPL 176A
          169A BSS
01155          TSX (SPH),4
01156          PZE 8)3N
               BSS
01157          TSX (FIL),4
01160 170A     CLA V-3
01161          FSB W-18
01162          STO TE3
01163 171A     LDQ V-4
01164          FMP 3)
01165          STO TE
01166 172A     LDQ V-5
01167          FMP 3)
01170          STO TE2
01171 173A     CLA W-69
01172          UFA 6)
01173          LRS
01174          ANA 6)+1
01175          LLS
01176          ALS 18
01177          STO INT
          174A BSS
01200          TSX (SPH),4
01201          PZE 8)3J
01202 175A     LDQ INT
01203          STR
01204          LDQ V-2
01205          STR
01206          LDQ V-1
01207          STR
```

```
01210          LDQ TE3              01300        OCT +000016000000    01372        BCD 1INT,4H
01211          STR                  01301        OCT +000017000000    01373 8)3I   BCD 1 (5HO
01212          LDQ TE1              01302        OCT +000020000000    01374 8)3H   BCD 1(2I6)
01213          STR                  01303        OCT +000021000000    01375        BCD 1EXIT)
01214          LDQ TE2              01304        OCT +000022000000    01376        BCD 1DY TO
01215          STR                  01305        OCT +000025000000    01377        BCD 1 - REA
01216          LDQ V-6              01306        OCT +000026000000    01400        BCD 1ONITOR
01217          STR                  01307        OCT +000011000000    01401        BCD 1 FOR M
01220          LDQ V-7              01310        OCT +000002000000    01402        BCD 1ITCHES
01221          STR                  01311        OCT +000001000000    01403        BCD 1NSE SW
01222          LDQ V-8              01312        OCT +000073000000    01404        BCD 1SET SE
01223          STR                  01313        OCT +000102000000    01405 8)3G   BCD 1(48HO
01224          LDQ W                01314 3)     OCT +206712273407    01406        BCD 1CUTE)
01225          STR                  01315        OCT +202622077326    01407        BCD 1NG EXE
01226          LDQ W-1              01316        OCT +000000000000    01410        BCD 1N DURI
01227          STR                  01317        OCT +201400000000    01411        BCD 1OSITIO
01230          LDQ W-27             01320        OCT +223444760000    01412        BCD 1N    P
01231          STR                  01321 6)     OCT +233000000000    01413        BCD 1CHES I
01232          LDQ W-65             01322        OCT +000000077777    01414        BCD 1E SWIT
01233          STR                  01323        OCT +000000000000    01415        BCD 1L SENS
               BSS                  01324        OCT +000001000000    01416        BCD 1SET AL
01234          TSX (FIL),4          01325        OCT +000000000000    01417 8)3F   BCD 1(54HO
      176A     BSS                  01326        BCD 1RO)             01420        BCD 1RED)
01235          TSX OUTPUT,4         01327        BCD 1 OR ZE          01421        BCD 1ONSIDE
01236 177A     PSE 116              01330        BCD 1GATIVE          01422        BCD 1EING C
01237          TRA E)R              01331        BCD 1 IS NE          01423        BCD 1BLEM B
01240          TRA 142A             01332        BCD 1 W(1)           01424        BCD 1OR PRO
01241 E)R      SXD C)G3,2           01333 8)3N   BCD 1(28HO           01425        BCD 1 SS2 F
01242 178A     TRA 178A+4,2         01334        BCD 1E9.2)           01426        BCD 1S1 AND
01243          TRA 181A             01335        BCD 1   ,1P3         01427        BCD 1 SET S
01244          TRA 180A             01336        BCD 19.2,2H          01430 8)3E   BCD 1 (46HO
01245 179A     CLA V-3              01337        BCD 1H ,1PE          01431        BCD 1)
01246          FSB W-39             01340        BCD 1E9.2.1          01432        BCD 1E14.7
01247 179A1    TZE 165A             01341        BCD 1    ,1P         01433 8)15   BCD 1  (I3,
01250          TPL 136A             01342        BCD 19.2,3H          01434        BCD 1)
01251          TRA 165A             01343        BCD 15,1P7E          01435 8)23   BCD 1 (12A6
01252 180A     CLA W-27             01344 8)3J   BCD 1(1HO,I
01253          FSB W-39             01345        BCD 1X)
01254 180A1    TZE 166A             01346        BCD 1,7X,1H
01255          TPL 136A             01347        BCD 1HRANGE
01256          TRA 166A             01350        BCD 1),5X,5
01257 181A     CLA W-59             01351        BCD 1,4HW(2
01260          FSB W-39             01352        BCD 1(1),6X
01261 181A1    TZE 166A             01353        BCD 15X,4HW
01262          TPL 136A             01354        BCD 1IGPHI,
01263          TRA 166A             01355        BCD 16X,6HS
01264 D)40V    LXD C)G1,4           01356        BCD 1THETA,
01265 182A     CLA KWIT             01357        BCD 1,8HSIG
01266          SUB 2)+12            01360        BCD 1IGR,3X
01267 182A1    TZE 24A              01361        BCD 15X,4HS
01270 E)10     SXD C)G3,2           01362        BCD 13HPHI,
01271 183A     CLA KWIT             01363        BCD 1TA,5X,
01272          SUB 2)+13            01364        BCD 1,5HTHE
01273 183A1    TZE 17A              01365        BCD 1-RO,3X
01274          TPL D)10J            01366        BCD 17HV(4)
01275          TRA D)10J            01367        BCD 12),4X,
01276 2)       OCT +000005000000    01370        BCD 1X,4HV(
01277          OCT +000015000000    01371        BCD 1DELV,5
```

B. FUNCTION SLANTR

This function is used to calculate the non-deviated ray path between the transmitter, R, and the starting point, S, (See Figures 1 and 2) from the given input data describing the problem under consideration. The input data should be designed in a manner that this assumption is true.

```
C     FUNCTION  SLANTR   DECEMBER 22,1960  IBM-7090
1     FUNCTION SLANTR(ARG7,ARG8,ARG12,ARG13)
2     TERM1= SQRTF ((1.0+ (ARG13/ARG12))**2 - ARG8**2)
3     SLANTR =  ARG12* (-ARG7+ TERM1)
      RETURN
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
              EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC
  1     1 00000            2     3 00022            3     4 00040

                               STORAGE NOT USED BY PROGRAM

      DEC   OCT                DEC   OCT
       55 00067              32531 77461

                               LOCATIONS OF NAMES IN TRANSFER VECTOR

      DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
SORT    0 00000

      STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

      DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
SLANTR 54 00066        TERM1    53 00065

                     STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

      DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
1)     50 00062        2)       43 00053        3)      44 00054        6)      45 00055

                        ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
SORT
```

```
00000 SQRT    BCD 1SQRT
00001 $       HTR
00002         HTR
00003         HTR
00004         BCD 1SLANTR
00005         SXD $,1
00006         SXD $+1,2
00007         SXD $+2,4
00010         CLA 1,4
00011         STA 3A+14
00012         CLA 2,4
00013         STA 3A
00014         STA 3A+1
00015         CLA 3,4
00016         STA 3A+4
00017         STA 3A+17
00020         CLA 4,4
00021         STA 3A+3
00022 3A      LDQ ARG8
00023         FMP ARG8
00024         STO 1)+1
00025         CLA ARG13
00026         FDP ARG12
00027         STQ 1)+2
00030         CLA 3)
00031         FAD 1)+2
00032         STO 1)
00033         LDQ 1)
00034         FMP 1)
00035         FSB 1)+1
              BSS
00036         TSX SQRT,4
00037         STO TERM1
00040 4A      CLS ARG7
00041         FAD TERM1
00042         STO 1)+1
00043         LDQ ARG12
00044         FMP 1)+1
00045         STO SLANTR
00046 5A      CLA SLANTR
00047         LXD $,1
00050         LXD $+1,2
00051         LXD $+2,4
00052         TRA 5,4
00053 2)      OCT +000002000000
00054 3)      OCT +201400000000
00055 6)      OCT +233000000000
00056         OCT +000000077777
00057         OCT +000000000000
00060         OCT +000001000000
00061         OCT +000000000000
```

C. FUNCTION QATAN

Function QATAN permits the evaluation of the value of the arctangent of an angle with proper quadrant allocation.

```
C     FUNCTION QATAN   DECEMBER 22,1960  IBM-7090
1     FUNCTION QATAN(X,Y)
      PI = 3.1415927
      IF(ABSF(X)-1.0E-18)9,2,2
9     IF(ABSF(Y)-1.0E18)2,2,17
17    QATAN = PI/2.0
      GO TO 6
2     ARG = Y/X
3     IF(ARG) 4,13,5
4     ARG = -ARG
5     QATAN  = ATANF(ARG)
6     IF(X) 8,7,7
7     IF(Y) 10,18,18
8     IF(Y) 12,11,11
10    QATAN =(2.0*PI) - QATAN
      GO TO 18
11    QATAN = PI - QATAN
      GO TO 18
12    QATAN = PI + QATAN
      GO TO 18
13    IF(X)  14,16,16
14    QATAN = PI
15    GO TO 18
16    QATAN = 0.0
18    RETURN
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	1	00000	9	5	00031	17	6	00037	2	8	00043	3	9	00046
4	10	00051	5	11	00053	6	12	00056	7	13	00062	8	14	00066
10	15	00072	11	17	00077	12	19	00103	13	21	00107	14	22	00112
15	23	00114	16	24	00115	18	25	00117						

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
98	00142	32561	77461

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
ATAN	0	00000								

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT	DEC	OCT	DEC	OCT
ARG	97	00141	PI	96	00140	QATAN	95	00137				

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT	DEC	OCT	DEC	OCT
2)	84	00124	3)	85	00125	6)	90	00132				

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

ATAN

```
00000 ATAN   BCD 1ATAN
00001 $      HTR
00002        HTR
00003        HTR
00004        BCD 1QATAN
00005        SXD $,1
00006        SXD $+1,2
00007        SXD $+2,4
00010        CLA 1,4
00011        STA 3A+2
00012        STA 3A+18
00013        STA 3A+28
00014        STA 3A+53
00015        CLA 2,4
00016        STA 3A+7
00017        STA 3A+17
00020        STA 3A+32
00021        STA 3A+36
00022 3A     CLA 3)
00023        STO PI
00024 4A     CLA X
00025        SSP
00026        FSB 3)+1
00027 4A1    TZE 8A
00030        TPL 8A
00031 5A     CLA Y
00032        SSP
00033        FSB 3)+2
00034 5A1    TZE 8A
00035        TPL 6A
00036        TRA 8A
00037 6A     CLA PI
00040        FDP 3)+3
00041        STQ QATAN
00042 7A     TRA 12A
00043 8A     CLA Y
00044        FDP X
00045        STQ ARG
00046 9A     CLA ARG
00047 9A1    TZE 21A
00050        TPL 11A
00051 10A    CLS ARG
00052        STO ARG
00053 11A    CLA ARG
             BSS
00054        TSX ATAN,4
00055        STO QATAN
00056 12A    CLA X
00057 12A1   TZE 13A
00060        TPL 13A
00061        TRA 14A
00062 13A    CLA Y
00063 13A1   TZE 25A
00064        TPL 25A
00065        TRA 15A
00066 14A    CLA Y
00067 14A1   TZE 17A
00070        TPL 17A
00071        TRA 19A
00072 15A    LDQ 3)+3
00073        FMP PI
00074        FSB QATAN
00075        STO QATAN
00076 16A    TRA 25A
00077 17A    CLA PI
00100        FSB QATAN
00101        STO QATAN
00102 18A    TRA 25A
00103 19A    CLA PI
00104        FAD QATAN
00105        STO QATAN
00106 20A    TRA 25A
00107 21A    CLA X
00110 21A1   TZE 24A
00111        TPL 24A
00112 22A    CLA PI
00113        STO QATAN
00114 23A    TRA 25A
00115 24A    CLA 3)+4
00116        STO QATAN
00117 25A    CLA QATAN
00120        LXD $,1
00121        LXD $+1,2
00122        LXD $+2,4
00123        TRA 3,4
00124 2)     OCT +000000000000
00125 3)     OCT +202622077326
00126        OCT +105447113564
00127        OCT +274674055531
00130        OCT +202400000000
00131        OCT +000000000000
00132 6)     OCT +233000000000
00133        OCT +000000077777
00134        OCT +000000000000
00135        OCT +000001000000
00136        OCT +000000000000
```

D. FUNCTION ARCOS

Function ARCOS permits the evaluation of the value of the arccosine of an angle. Presently no quadrant allocation is made.

```
C      FUNCTION ARCOS  DECEMBER 22, 1960      IBM-7090
       FUNCTION  ARCOS(Q)
 1     IF(ABSF(Q)-1.0E-15)2,2,3
 2     ARCOS =1.5707963
       GO TO 8
 3     B1=  Q*Q
 4     B2=   SQRTF(1.0/B1 -1.0)
 5     ARCOS =  ATANF(B2)
 6     IF(Q)7,8,8
 7     ARCOS = 3.1415927 - ARCOS
 8     RETURN
       END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
        EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN  IFN   LOC        EFN  IFN   LOC        EFN  IFN   LOC        EFN  IFN   LOC        EFN  IFN   LOC
  1    3 00016          2    4 00023          3    6 00026          4    7 00031          5    8 00037
  6    9 00042          7   10 00045          8   11 00050

                                 STORAGE NOT USED BY PROGRAM

     DEC   OCT                DEC   OCT
      58 00072              32561 77461

                          LOCATIONS OF NAMES IN TRANSFER VECTOR

     DEC   OCT                DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
ATAN   1 00001        SQRT      0 00000

       STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

     DEC   OCT                DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
  B2  57 00071          B1     56 00070     ARCOS    55 00067

                  STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

     DEC   OCT                DEC   OCT               DEC   OCT               DEC   OCT               DEC   OCT
 2)   45 00055          3)     46 00056        6)    50 00062

                        ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
SQRT      ATAN
```

```
00000 SQRT    BCD 1SQRT
00001 ATAN    BCD 1ATAN
00002 $       HTR
00003         HTR
00004         HTR
00005         BCD 1ARCOS
00006         SXD $,1
00007         SXD $+1,2
00010         SXD $+2,4
00011         CLA 1,4
00012         STA 3A
00013         STA 3A+8
00014         STA 3A+9
00015         STA 3A+20
00016 3A      CLA Q
00017         SSP
00020         FSB 3D
00021 3A1     TZE 4A
00022         TPL 5A
00023 4A      CLA 3D+1
00024         STO ARCOS
00025 5A      TRA 11A
00026 6A      LDQ Q
00027         FMP Q
00030         STO B1
00031 7A      CLA 3D+2
00032         FDP B1
00033         XCA
00034         FSB 3D+2
              BSS
00035         TSX SQRT,4
00036         STO S2
00037 8A      CLA S2
              BSS
00040         TSX ATAN,4
00041         STO ARCOS
00042 9A      CLA Q
00043 9A1     TZE 11A
00044         TPL 11A
00045 10A     CLA 3D+3
00046         FSB ARCOS
00047         STO ARCOS
00050 11A     CLA ARCOS
00051         LXD $,1
00052         LXD $+1,2
00053         LXD $+2,4
00054         TRA 2,4
00055 2D      OCT +000000000000
00056 3D      OCT +113715126246
00057         OCT +201622077323
00060         OCT +201400000000
00061         OCT +202622077326
00062 6D      OCT +233000000000
00063         OCT +000000077777
00064         OCT +000000000000
00065         OCT +000001000000
00066         OCT +000000000000
```

E. SUBROUTINE COORD

This subroutine is used to transform the problem from the radar system to an earth centered geomagnetic spherical coordinate system.

```
0     SUBROUTINE COORD  DECEMBER 22,1960   IBM-7090
1    CSUBROUTINE  COORD(X,Y,Z,ARG1,ARG2,ARG3,ARG4,ARG5,ARG6,ARG7,ARG8,
     1ARG9,ARG10,ARG11,ARG12)
2    DX=((ARG2*ARG3*ARG6-ARG4*ARG5)-ARG11*ARG7+ARG1*ARG3*ARG11*ARG8*ARG9
     1-(ARG2*ARG3*ARG5+ ARG4*ARG6)*ARG11*ARG3*ARG10 +ARG12*(ARG2*ARG3*
     2ARG3-ARG4*ARG5))
3    DY  (-ARG3*ARG6*ARG11*ARG7 +ARG2*ARG11*ARG8*ARG9+ARG1+ARG5*ARG11
     1*ARG3*ARG10-ARG12*ARG1*ARG6)
4    DZ= ((ARG2*ARG4*ARG6+ARG3*ARG5)*ARG11*ARG7+ ARG1*ARG4+ARG11*ARG8
     1*ARG9+(-ARG2*ARG4*ARG5+ARG3*ARG6)*ARG11*ARG3*ARG10 +ARG12 *(ARG2
     2*ARG4*ARG5 +ARG3*ARG5))
5     RETURN
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
 E.TERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNA  FORMU A NUMBERS AND OCTAL LOCATIONS
EFN   IFN   'OC        EF    IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC
       1 00000           2     7 00123          3     4 00200          4     5 00240          5     6 00316

                       STORAGE NOT USED BY PROGRAM

DEC   OCT              DEC   OCT
224 00340            3 561 77 61

             STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
21. 00330     2)       212 00322      6)      211 00323
```

```
00000 $     HTR
00001       HTR
00002       HTR
00003       BCD 1000PD
00004       SXD $,1
00005       SXD $+1,2
00006       SXD $+2,4
00007       CLA 1,4
00010       STA 3A+44
00011       CLA 2,4
00012       STA 3A+76
00013       CLA 3,4
00014       STA 3A+122
00015       CLA 4,4
00016       STA 3A+29
00017       STA 3A+48
00020       STA 3A+51
00021       STA 3A+67
00022       STA 3A+107
00023       CLA 5,4
00024       STA 3A+4
00025       STA 3A+16
00026       STA 3A+60
00027       STA 3A+81
00030       STA 3A+93
00031       CLA 6,4
00032       STA 3A+6
00033       STA 3A+19
00034       STA 3A+31
00035       STA 3A+77
00036       STA 3A+89
00037       CLA 7,4
00040       STA 3A
00041       STA 3A+12
00042       STA 3A+83
00043       STA 3A+95
00044       STA 3A+109
00045       CLA 8,4
00046       STA 3A+1
00047       STA 3A+15
00050       STA 3A+53
00051       STA 3A+78
00052       STA 3A+92
00053       CLA 9,4
00054       STA 3A+3
00055       STA 3A+13
00056       STA 3A+45
00057       STA 3A+69
00060       STA 3A+80
00061       STA 3A+90
00062       CLA 10,4
00063       STA 3A+37
00064       STA 3A+66
00065       STA 3A+115
00066       CLA 11,4
00067       STA 3A+26
00070       STA 3A+35
00071       STA 3A+57
00072       STA 3A-64
00073       STA 3A+104
00074       STA 3A+113
00075       CLA 12,4
00076       STA 3A+28
00077       STA 3A+59
00100       STA 3A+106
00101       CLA 13,4
00102       STA 3A+21
00103       STA 3A+50
00104       STA 3A+99
00105       CLA 14,4
00106       STA 3A+24
00107       STA 3A+33
00110       STA 3A+40
00111       STA 3A-55
00112       STA 3A+62
00113       STA 3A+71
00114       STA 3A+102
00115       STA 3A+111
00116       STA 3A+118
00117       CLA 15,4
00120       STA 3A+9
00121       STA 3A+46
00122       STA 3A+86
00123 3A    LDQ ARG4
00124       FMP ARG5
00125       STO 1)+1
00126       LDQ ARG6
00127       FMP ARG2
00130       XCA
00131       FMP ARG3
00132       FSB 1)+1
00133       STO 1)+2
00134       LDQ ARG12
00135       FMP 1)+2
00136       STO 1)+3
00137       LDQ ARG4
00140       FMP ARG6
00141       STO 1)+4
00142       LDQ ARG5
00143       FMP ARG2
00144       XCA
00145       FMP ARG3
00146       FAD 1)+4
00147       STO 1)+5
00150       LDQ ARG10
00151       FMP 1)+5
00152       XCA
00153       FMP ARG11
00154       XCA
00155       FMP ARG3
00156       STO 1)+6
00157       LDQ ARG9
00160       FMP ARG1
00161       XCA
00162       FMP ARG3
00163       XCA
00164       FMP ARG11
00165       XCA
00166       FMP ARG8
00167       STO 1)+7
00170       LDQ ARG7
00171       FMP 1)+2
00172       XCA
00173       FMP ARG11
00174       FAD 1)+7
00175       FSB 1)+6
00176       FAD 1)+3
00177       STO X
00200 4A    LDQ ARG6
00201       FMP ARG12
00202       XCA
00203       FMP ARG1
00204       STO 1)+1
00205       LDQ ARG10
00206       FMP ARG1
00207       XCA
00210       FMP ARG5
00211       XCA
00212       FMP ARG11
00213       XCA
00214       FMP ARG8
00215       STO 1)+2
00216       LDQ ARG9
00217       FMP ARG2
00220       XCA
00221       FMP ARG11
00222       XCA
00223       FMP ARG8
00224       STO 1)+3
00225       LDQ ARG7
00226       FMP ARG1
00227       XCA
00230       FMP ARG6
00231       XCA
00232       FMP ARG11
00233       CHS
00234       FAD 1)+3
00235       FAD 1)+2
00236       FSB 1)+1
00237       STO Y
00240 5A    LDQ ARG3
00241       FMP ARG5
00242       STO 1)+1
00243       LDQ ARG6
00244       FMP ARG2
00245       XCA
00246       FMP ARG4
00247       FAD 1)+1
00250       STO 1)+2
00251       LDQ ARG12
00252       FMP 1)+2
00253       STO 1)+3
00254       LDQ ARG3
00255       FMP ARG8
```

```
00256        STO 1D+4
00257        LDQ ARG5
00260        FMP ARG2
00261        XCA
00262        FMP ARG4
00263        CHS
00264        FAD 1D+4
00265        STO 1D+5
00266        LDQ ARG10
00267        FMP 1D+5
00270        XCA
00271        FMP ARG11
00272        XCA
00273        FMP ARG8
00274        STO 1D+6
00275        LDQ ARG9
00276        FMP ARG1
00277        XCA
00300        FMP ARG4
00301        XCA
00302        FMP ARG11
00303        XCA
00304        FMP ARG8
00305        STO 1D+7
00306        LDQ ARG7
00307        FMP 1D+2
00310        XCA
00311        FMP ARG11
00312        FAD 1D+7
00313        FAD 1D+6
00314        FAD 1D+3
00315        STO Z
00316 6A     LXD $,1
00317        LXD $+1,2
00320        LXD $+2,4
00321        TRA 16,4
00322 20     OCT +000000000000
00323 60     OCT +233000000000
00324        OCT +000000077777
00325        OCT +000000000000
00326        OCT +000001000000
00327        OCT +000000000000
```

F. SUBROUTINE DAUX

Subroutine DAUX is used to define the differential equations that are to be numerically integrated. As a result the previously described ray trace equations are defined in this subroutine.

```
C     SUBROUTINE DAUX    DECEMBER 22,1960     IBM-7090
 1    SUBROUTINE  DAUX
      DIMENSION RECORD(12),W(11),V(25)
      COMMON RECORD,V,W,N
      CALL  FINDEX
 4    TER1 = W(1)*W(1)
 5    TER2 = SINF(V(5))
 6    TER3 =COSF(V(5))
 7    IF(ABSF(TER2)-1.0E-8)17,8,8
 17   TER2 = 1.0E-8
 8    V(13) = (1.0/TER1) *(V(7) - W(1)*W(4))
 9    V(14) = (1.0/(V(4)*TER1)) *(V(8) - W(1)*W(5))
 10   V(15) = (1.0/(V(4)*TER1*TER2))*(V(9)- W(1)* W(6))
 11   V(16) = (1.0/W(1))* W(7) + V(3)* V(14) +V(9)* V(15)* TER2
 12  DV(17) = (1.0/V(4))*( (1.0/W(1)) * W(8)- V(8)* V(13) + V(4)* V(9)*
     1 V(15)* TER3 )
 13  DV(18) = (1.0/(V(4)* TER2))*((1.0/ W(1))* W(9)- V(9)* V(13)* TER2
     1 - V(4)* V(9)* V(14)* TER3)
 14   V(19) =((1.0/TER1)*(SQRTF ( TER1        + W(10)**2  )))
 15   V(20) = (1.0/ 3.0E5 )*(1.0 + (W(3)/W(1))* W(11) )
 16   V(21) =  -(W(3)/3.0E5 ) *( W(2)/ W(1))
 20   RETURN
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                     STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

     DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
 N 32139 76874      RECORD 32561 77461         U 32549 77445         W 32438 77266

            EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

 EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC
   1     1 00000         4     6 00014         5     7 00017         6     8 00022         7     9 00025
  17    10 00032         8    11 00034         9    12 00045        10    13 00061        11    14 00077
  12    15 00115        13    16 00141        14    17 00172        15    18 00203        16    19 00214
  20    20 00224

                                      STORAGE NOT USED BY PROGRAM

     DEC   OCT             DEC   OCT
     169 00251           32187 76673

                                 LOCATIONS OF NAMES IN TRANSFER VECTOR

     DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
 COS   2 00002     RINDEX    0 00000     SIN     1 00001     SQRT      3 00003

        STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

     DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
TER1 168 00250      TER2   167 00247      TER3   166 00246

                          STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

     DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
1)   161 00241      2)     152 00230      3)     153 00231      6)     156 00234

                              ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
RINDEX        SIN         COS         SQRT
```

```
00000 RINDEX BCD 1RINDEX
00001 SIN    BCD 1SIN
00002 COS    BCD 1COS
00003 SQRT   BCD 1SQRT
00004 $      HTR
00005        HTR
00006        HTR
00007        BCD 1DAUX
00010        SXD $,1
00011        SXD $+1,2
00012        SXD $+2,4
      5A     BSS
00013        TSX RINDEX,4
00014 6A     LDQ W
00015        FMP W
00016        STO TER1
00017 7A     CLA V-4
             BSS
00020        TSX SIN,4
00021        STO TER2
00022 8A     CLA V-4
             BSS
00023        TSX COS,4
00024        STO TER3
00025 9A     CLA TER2
00026        SSP
00027        FSB 3)
00030 9A1    TZE 11A
00031        TPL 11B
00032 10A    CLA 3)
00033        STO TER2
00034 11A    LDQ W
00035        FMP W-3
00036        CHS
00037        FAD V-6
00040        STO 1)+1
00041        CLA 3)+1
00042        FDP TER1
00043        FMP 1)+1
00044        STO V-12
00045 12A    LDQ W
00046        FMP W-4
00047        CHS
00050        FAD V-7
00051        STO 1)+1
00052        LDQ V-3
00053        FMP TER1
00054        STO 1)+2
00055        CLA 3)+1
00056        FDP 1)+2
00057        FMP 1)+1
00060        STO V-13
00061 13A    LDQ W
00062        FMP W-5
00063        CHS
00064        FAD V-8
00065        STO 1)+1
00066        LDQ TER2

00067        FMP V-3
00070        XCA
00071        FMP TER1
00072        STO 1)+2
00073        CLA 3)+1
00074        FDP 1)+2
00075        FMP 1)+1
00076        STO V-14
00077 14A    LDQ TER2
00100        FMP V-8
00101        XCA
00102        FMP V-14
00103        STO 1)+1
00104        LDQ V-7
00105        FMP V-13
00106        STO 1)+2
00107        CLA 3)+1
00110        FDP W
00111        FMP W-6
00112        FAD 1)+2
00113        FAD 1)+1
00114        STO V-15
00115 15A    LDQ TER3
00116        FMP V-3
00117        XCA
00120        FMP V-8
00121        XCA
00122        FMP V-14
00123        STO 1)+1
00124        LDQ V-7
00125        FMP V-12
00126        STO 1)+2
00127        CLA 3)+1
00130        FDP W
00131        FMP W-7
00132        FSB 1)+2
00133        FAD 1)+1
00134        STO 1)+3
00135        CLA 3)+1
00136        FDP V-3
00137        FMP 1)+3
00140        STO V-16
00141 16A    LDQ TER3
00142        FMP V-3
00143        XCA
00144        FMP V-8
00145        XCA
00146        FMP V-13
00147        STO 1)+1
00150        LDQ TER2
00151        FMP V-8
00152        XCA
00153        FMP V-12
00154        STO 1)+2
00155        CLA 3)+1
00156        FDP W
00157        FMP W-8
00160        FSB 1)+2

00161        FSB 1)+1
00162        STO 1)+3
00163        LDQ V-3
00164        FMP TER2
00165        STO 1)+4
00166        CLA 3)+1
00167        FDP 1)+4
00170        FMP 1)+3
00171        STO V-17
00172 17A    LDQ W-9
00173        FMP W-9
00174        FAD TER1
             BSS
00175        TSX SQRT,4
00176        STO 1)+1
00177        CLA 3)+1
00200        FDP TER1
00201        FMP 1)+1
00202        STO V-18
00203 18A    CLA W-2
00204        FDP W
00205        FMP W-10
00206        FAD 3)+1
00207        STO 1)+1
00210        CLA 3)+1
00211        FDP 3)+2
00212        FMP 1)+1
00213        STO V-19
00214 19A    CLA W-1
00215        FDP W
00216        STO 1)+1
00217        CLA W-2
00220        FDP 3)+2
00221        FMP 1)+1
00222        CHS
00223        STO V-20
00224 20A    LXD $,1
00225        LXD $+1,2
00226        LXD $+2,4
00227        TRA 1,4
00230 2)     OCT +000002000000
00231 3)     OCT +146527461670
00232        OCT +201400000000
00233        OCT +223444760000
00234 6)     OCT +233000000000
00235        OCT +000000077777
00236        OCT +000000000000
00237        OCT +000001000000
00240        OCT +000000000000
```

G. SUBROUTINES INT AND INTM

This is a generally available SHARE program which permits the numerical integration of a chosen set of first order non-linear differential equations. It can be operated in three possible numerical integration modes (a) Runge-Kutta with a fixed integration mesh size, (b) fourth order Adams-Moulton with a fixed integration mesh size, and (c) fourth order Adams-Moulton with a variable integration mesh size that is controlled by an error sensing routine. Because no FORTRAN source program listing is available, a SHARE description of this routine is given along with a SAP listing.

IDENTIFICATION

RW INT, Adams-Moulton, Runge-Kutta Integration
704 - FORTRAN SAP Language Subroutine
Space Technology Laboratories,
Robert Causey and Werner L. Frank,
November 30, 1958

ABSTRACT

FORTRAN version of RW-DE2F which integrates a system of N simultaneous, first order, ordinary differential equations. Option of using either 4th order Runge-Kutta method or 4th order predictor-corrector method (Adams-Moulton) is provided. Also option of automatic error control with variable step-size is provided. Input and output are single precision but double precision is used internally to control round-off errors. Requires 12N + 3 cells for data and 693 words for program.

PURPOSE

This FORTRAN subprogram integrates a set of N simultaneous, first order differential equations. It is the FORTRAN version of the standard subroutine RW-DE2F.

RESTRICTIONS

This program has two distinct entries, one for set up and the second for performing the integration steps. The user must supply a FORTRAN subprogram (with the name DAUX) which evaluates the derivatives y'.

METHOD

The user has the option of using either a fourth order Runge-Kutta method or the fourth order Adams-Moulton method with a fixed step-size. There is also a variable step-size mode.

While input and output to this routine are single precision, double precision is used internally to control round-off errors. Truncation error is controlled either by choosing an appropriate step-size, or by using the variable step-size mode of operation.

For details of the method see RW-DE2F.

USAGE

a. Calling Sequence for set up (performed prior to initiating the integration).

CALL INT (V, N, A1, A2, A3, A4, A5, A6, A7)

Where V is a region of at least dimension 12N + 3

N is the number of equations

A1 is the option word

A2 is $\overline{E}$

A3 is M

A4 is A

A5 is h_{max}

A6 is h_{min}

A7 is β

For meaning of A1 - A7 see Appendix A and B of RW-DE2F.

Region V contains the following information prior to Set Up entry.

V(2) = x, initial value of independent variable
V(3) = h, value of step-size

$$\left.\begin{array}{l} V(4) = y_1 \\ \vdots \\ V(3+N) = y_N \end{array}\right\} \text{values of dependent variables } y_i$$

$$\left.\begin{array}{l} V(4+N) = y_1' \\ \vdots \\ V(3+2N) = y_N' \end{array}\right\}$$ values of the derivatives y_i to be supplied by the auxiliary DAUX.

Note: This region and the parameter N should be placed in COMMON since it is necessarily referred to in the main program and in the auxiliary. The cell V (1) is set up by the subprogram RW INT and will contain N scaled at 35.

b. Calling Sequence for integrating one step.

CALL INTM

No arguments are required for this statement.

SPACE REQUIRED

693 cells

CHECKOUT

This routine has been extensively tested on several check problems. In all cases the errors were approximately equal to their expected values, and there were no indications that round-off errors accumulate rapidly.

METHOD

References:

1. S. D. Conte and J. Titus, An interpretive floating point subroutine for the solution of systems of ordinary differential equations, Appendix I, Proc. Math. Committee of Univac Scientific Exchange Meeting, Nov. 21-22, 1957 (Obtainable from Remington Rand Univac, St. Paul, Minnesota).

2. E. K. Blum, A modification of the Runge-Kutta fourth-order method, Appendix H, Proc. Math. Committee of Univac Scientific Exchange Meeting, Nov. 21-22, 1957.

In this routine the user is allowed an option of using either the Runge-Kutta classical fourth-order method as modified by E. K. Blum [Ref. (2)] or the Adams-Moulton predictor-corrector method using the Runge-Kutta method for starting the process. Let the system of equations to be solved be given in the form

$$\text{(1)} \quad \left.\begin{aligned} y_i' &= f_i\left(x, y_1, y_2, \ldots, y_N\right) \\ y_i\left(x_o\right) &= y_{io} \end{aligned}\right\} \quad i = 1, 2, \ldots, N.$$

Let y_{in} be the value of y_i at $x = x_n$ and f_{in} the derivation of y_i at $x = x_n$ and let h be the increment (step-size) of the independent variable x. The classical Runge-Kutta fourth-order method uses the formulas

$$\begin{aligned} k_{i1} &= h f_i\left(x_n, y_{in}\right) \quad , \\ k_{i2} &= h f_i\left(x_n + \frac{1}{2}h , y_{in} + \frac{1}{2}k_{i1}\right) \quad , \\ k_{i3} &= h f_i\left(x_n + \frac{1}{2}h , y_{in} + \frac{1}{2}k_{i2}\right) \quad , \end{aligned}$$

$$
\begin{aligned}
k_{i4} &= h f_i \left(x_n + h\,,\; y_{in} + k_{i3}\right) \,, \\
(2) \qquad y_{i,\,n+1} &= y_n + \frac{1}{6}\left(k_{i1} + 2k_{i2} + 2k_{i3} + k_{i4}\right) \,,
\end{aligned}
$$

The following formulas (we omit the subscript i for notational simplicity) were derived by E. K. Blum to control the growth of round-off errors.

$$
(3) \qquad \begin{cases} z_o = y_n \,, \\ q_o = q_{4n} \end{cases}
$$

$$
(4) \qquad \begin{cases} P_o = h\, f\left(x_n\,,\; z_o\right) \\ r_1 = L^{(1)} R^{(1)} \left[\frac{1}{2} P_o - q_o\right] \,; \\ z_1 = z_o + r_1 \,, \\ q_1 = 3\, r_1 - \left[\frac{1}{2} P_o - q_o\right] \,, \end{cases}
$$

$$
(5) \qquad \begin{cases} P_1 = h\, f(x_n + \frac{1}{2} h\,,\; z_1) \,, \\ r_2 = L^{(2)} R^{(2)} \left[\frac{1}{2} P_1 - \frac{1}{2} q_1\right] \,, \\ z_2 = z_1 + r_2 \,, \\ q_2 = - r_2 - \frac{1}{3} q_1 + \frac{1}{2} P_1 \,, \end{cases}
$$

$$
(6) \qquad \begin{cases} P_2 = h\, f(x_n + \frac{1}{2} h\,,\; z_2 \,, \\ r_3 = L^{(3)} R^{(3)} \left[P_2\right] \,, \\ z_3 = z_2 + r_3 \,, \\ q_3 = - r_3 + q_2 \,, \end{cases}
$$

$$
(7)\quad \begin{cases} P_3 & = h f\left(x_n + h,\ z_3\right) + 2P_2\ , \\ r_4 & = L^{(4)} R^{(4)} \left[\frac{1}{6} P_3 + q_3\right]\ , \\ y_{n+1} & = z_4 = z_3 + r_4\ , \\ q_{4,\, n+1} & = 3\left[r_4 - \left(\frac{1}{6} P_3 + q_3\right)\right]\ , \end{cases}
$$

where $R^{(m)}$, $L^{(m)}$ denote operators which shift right m places or left m places respectively and q_{40} is taken to be zero to start the computation. (See Ref (2) for a complete description of this method.) Formulas (3) - (7) are those used in this routine.

The Adams-Moulton predictor-corrector formulas for the system (1) are

$$
(8)\quad y^{(p)}_{i,\, n+1} = y_{in} + \frac{h}{24}\left(55 f_{in} - 59 f_{i,\, n-1} + 37 f_{i,\, n-2} - 9 f_{i,\, n-3}\right)\ ,
$$

$$
(9)\quad y^{(c)}_{i,\, n+1} = y_{in} + \frac{h}{24}\left(9 f^{(p)}_{i,\, n+1} + 19 f_{in} - 5 f_{i,\, n-1} + f_{i,\, n-2}\right)\ .
$$

The corrector formular (9) is applied only once so that only two derivative evaluations are needed for each Adams-Moulton integration step. The starting values needed in (8) are obtained using the Runge-Kutta-Blum (RKB) method.

The Adams-Moulton method may be used either with a fixed step-size or with a variable step-size. The step-size to be used in the variable mode is determined as follows. Let

$$
(10)\quad \begin{aligned} E_{n+1} &= \operatorname*{Max}_{i} \frac{\left|y^{(p)}_{i,\, n+1} - y^{(c)}_{i,\, n+1}\right|}{14 D_i}\ , \\ D_i &= \operatorname{Max}\left\{\left|y^{(c)}_{i,\, n+1}\right|\ ,\ A\right\}\ , \end{aligned}
$$

where $A > 0$. The user will specify an upper bound $\bar{E}$ on the truncation error estimate E_{n+1}. This is equivalent to specifying the number of significant figures which the user desires to preserve locally throughout the integration. There must also be specified a constant $M > 0$

from which a lower bound $\underline{E} = M^{-1}\,\overline{E}$ is obtained. M should normally range from 50 to 150. The interval will then be decreased, left as it is, or increased according as the following inequalities hold:

(11a) If $E_{n+1} \geqslant \overline{E}$, the interval is reduced to βh $(0 < \beta < 1$

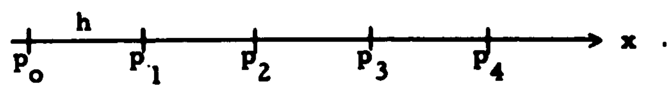
(11b) If $\underline{E} \leqslant E_{n+1} < \overline{E}$, the interval size is kept fixed.

(11c) If $E_{n+1} < \underline{E}$ for 3 successive steps, the step-size is increased to $\frac{1}{\beta}$ h.

Normally, the routine will take $\beta = 1/2$, unless β is otherwise specified. The constant A in (10) is used to prevent unnecessary reductions in $|h|$ whenever $|y_{i,\,n+1}|$ is small. Normally the routine will set A = 1. However, some other value for A may be specified by the user if he desires to use some other characteristic length for A. While the test based on (10) will guarantee that the local error does not exceed $\overline{E}$, the cumulative error will usually exceed $\overline{E}$. Hence, $\overline{E}$ should be chosen small enough to allow for an accumulation of truncation error. Normally $\overline{E}$ should be in the range $10^{-8} \leqslant \overline{E} \leqslant 10^{-3}$.

After an interval is increased, the program prevents increasing again until 6 more points have been completed. However, the program may decrease the interval as often as necessary.

Starting values for the Adams-Moulton formulas are always obtained using the RKB method whenever the interval size is changed, just as at the beginning of an integration. Consider the following diagram of the axis of the independent variable x



If the values of the y_i are computed at the points p_1, p_2, and p_3 using the RKB method and the truncation error test (11) calls for decreasing $|h|$ at the point p_4, then the routine returns to the point p_o and again computes three new points with the RKB method using the decreased value of $|h|$. If on the other hand (11a) holds at p_4 and the y_i at p_3 had been computed using the AM method, then the routine returns to the point p_3 for a new start. If the inequality in

(11c) is <u>not</u> satisfied at p_1, but <u>is</u> satisfied at p_2, p_3, and p_4, then a new start is initiated at p_4 with the increased value of $|h|$.

The user must provide a starting value for h and he may, if desired, specify a maximum value of $|h|$ beyond which the routine will not increase $|h|$ and a minimum value of $|h|$ below which it will not decrease $|h|$. Negative values of h may be supplied for backward integration.

Both the RKB method and the AM method incorporate round-off control features. This is performed in the RKB method by carrying the q's in formula (3)-(7). In the AM method this is done by keeping the y_{in} in double precision forming the sums $y_{in} + \nabla y_{in}$ in (8) and (9) in double precision. The derivative calculations are all performed in single precision. Both procedures have shown to be very effective in controlling the growth of round-off errors.

USAGE AND CODING INFORMATION (APPENDIX B)

There are two entries to this routine. The first must be used once at the beginning to set up the routine for integration of a given set o N differential equations. The second entry may be used any number of times after the first to integrate all y_i from x to x + h. The first entry has the following calling sequence.

Loc.	Instruction	Comment
A1-2	TSX DE2F, 4	Setup entry
A1-1	PZE T, 0, V	Parameter word with addresses
A1	(Binary integer)	Option word (= 0 or 1 or 2)
A2	(Floating-point number)	$\overline{E}$ Truncation error testing information
A3	(Floating-point number)	M Truncation error testing information
A4	(Floating-point number)	A
A5	(Floating-point number)	h_{max} Bounds on h, if any
A6	(Floating-point number)	h_{min} Bounds on h, if any
A7	(Floating-point number)	β - Increase or decrease h factor
A7 + 1	Return	

The eight parameter words have the following meaning, (A1-1): V is the address of the first word of a block of 12N + 3 cells, reserved by the user, with the arrangement

Loc.	Contents	
V	N	Fixed point binary integer, point at right
V + 1	x	Value of independent variable in floating point
V + 2	h	Value of step size in floating point
V + 3	y_1	Values of the y_i in floating point
V + 4	y_2	
.	.	
.	.	
.	.	
V + N + 2	y_N	

Loc.	Contents	
V + N + 3	y_1'	Locations where the user's auxiliary subroutine must place the derivatives y_1'.
V + N + 4	y_2'	
.		
.		
.		
V + 2N + 2	y_N'	
V + 2N + 3		10 N cells of temporary storage
etc.		

[Note: If the Runge-Kutta only option (see under A1 below) is used, it is only necessary to reserve a block of 4N + 3 cells].

Before executing the setup entry, the user must have already placed the appropriate numbers in cells V through V + N + 2.

The address V in the entry point of an auxiliary subroutine which the user must provide to evaluate the derivatives y_i' and store them in cells V + N + 3 through V + 2N + 2 as shown above. This auxiliary subroutine is entered by the calling sequence

Loc.	Instruction
A1-2	TSX V, 4
A1-1	Return

The setup entry uses the auxiliary subroutine to evaluate the derivatives for the initial data.

(A1): The option word may have any one of three values which designate three different modes of operation for RWINT

A1 = 0 designates the predictor-corrector variable h mode

A1 = 1 designates the fixed h Runge-Kutta only mode

A1 = 2 designates the fixed h predictor-corrector mode

For A1 = 1 or 2, the contents of A2 through A7 may be arbitrary.

(A2): This cell contains the upper bound $\bar{E} > 0$ for the truncation error testing done in the predictor-corrector variable h mode. $(10^{-8} \leq E \leq 10^{-3})$

(A3): This cell contains the number $M > 0$ from which the lower bound $\underline{E}$ is calculated. If $A3 = 0$, M is set equal to 100.

(A4): This cell contains the number $A > 0$ used to designate a fixed-point truncation error test as described in Appendix A. If $A4 = 0$, A is set equal to 1.

(A5): This cell may contain the upper bound $h_{max} > 0$ for $|h|$. If $A5 = 0$, this means that there is to be no upper bound for $|h|$.

(A6): This cell may contain the lower bound $h_{min} > 0$ for $|h|$. If $A6 = 0$, this means that there is to be no lower bound for $|h|$.

(A7): This cell may contain the factor $1 > \beta > 0$ used to increase or decrease $|h|$. If $A7 = 0$, β is set equal to 1/2.

The integration entry is quite simple and has the calling sequence

Loc.	Instruction
A1-2	TSX DE2F + 1, 4 Integration entry
A1-1	Return

Ordinarily, after execution of the integration entry, all y_i assume new values, x will have been advanced to the value $x + h$ and h will be unchanged. However, in the variable h mode, three other things can happen. (1) If the truncation error test indicates that $|h|$ should be increased, h will have been changed to $\beta^{-1}h$ unless $|\beta^{-1}h| > h_{max}$. If the truncation error test indicates that $|h|$ should be decreased, then h will have been changed to βh unless $|\beta h| < h_{min}$ and either (2) y_i and x will remain as they were before entry or (3) x will be changed to $x - 3h$ and the corresponding y_i values will occupy cells V + 3 through V + N + 2. Case (3) can only happen when successive decreases in $|h|$ are called for. On exit the values y_i' in V + N + 3 etc. are always those which correspond to the x and y_i in V + 1 and V + 3 etc.

The integration entry must be used for each integration step. In the variable h mode, a particular integration step may involve

either AM or RKB integration but not both. In the fixed h predictor-corrector mode, the first three integration entries involve RKB integration and all subsequent ones involve AM integration.

Whenever an integration step involves AM integration, the truncation error estimate E_{n+1} is in the accumulator on exit. Zero is always placed in the accumulator if the step involved RKB integration.

The setup entry may be used again at any time to set up another problem or to change the mode of operation.

In addition to the auxiliary subroutine for derivative evaluation and the 12N + 3 cells for data storage, the storage requirements are 693 words for RWINT plus 2 words of COMMON.

```
        ORG 0
DAUX    BCD 1DAUX
        HTR
        HTR
        HTR
INT     SXD INT-3,4
        SXD INT-2,2
        SXD INT-1,1
        CLA 1,4
        STA REV1+1
        STA REV1+2
        STA A2
        CLA 2,4
        STA A1
A1      CLA                     N
        ARS 18
A2      STO
        ALS 2
        STO C
        ALS 1
        ADD C                  -12N
        ADD C1                  12N+2
        STO C
        CLA 1,4
        SUB C
        STA PAR1
        CLA C
        ARS 1
        STO C
        ADD PAR1
        STA REV1
        STA REV1+3
        LXA C2,1
A4      CLA 3,4
        STA A5
A5      CLA
        STO PAR8+1,1
        TXI A6,4,-1
A6      TIX A4,1,1
        CLA PAR2
        LRS 18
        STO PAR2
        CLA DAUX
        STA AUX+2
        TSX REV,4
        TSX DE2F,4
PAR1    PZE 0,0,AUX
PAR2    PZE
PAR3    PZE
PAR4    PZE
PAR5    PZE
PAR6    PZE
PAR7    PZE
```

```
PAR8  PZE                                                        INT 0054
      TSX REV,4                                                  INT 0055
      LXD INT-2,2                                                INT 0056
      LXD INT-1,1                                                INT 0057
      LXD INT-3,4                                                INT 0058
      TRA 10,4                                                   INT 0059
INTM  SXD INT-3,4                                                INT 0060
      SXD INT-2,2                                                INT 0061
      SXD INT-1,1                                                INT 0062
      TSX REV,4                                                  INT 0063
      TSX DE2F+1,4                                               INT 0064
      TSX REV,4                                                  INT 0065
      LXD INT-3,4                                                INT 0066
      LXD INT-2,2                                                INT 0067
      LXD INT-1,1                                                INT 0068
      TRA 1,4                                                    INT 0069
REV   LXA C,1                                                    INT 0070
      LXD C1,2                                                   INT 0071
REV1  CLA 0,1                                                    INT 0072
      LDQ 0,2                                                    INT 0073
      STO 0,2                                                    INT 0074
      STQ 0,1                                                    INT 0075
      TXI REV3,2,1                                               INT 0076
REV3  TIX REV1,1,1                                               INT 0077
      TRA 1,4                                                    INT 0078
AUX   SXD C3,4                                                   INT 0079
      TSX REV,4                                                  INT 0080
      TSX 0,4                                                    INT 0081
      TSX REV,4                                                  INT 0082
      LXD C3,4                                                   INT 0083
      TRA 1,4                                                    INT 0084
C     PZE                                                        INT 0085
C1    DEC 2                                                      INT 0086
C2    DEC 7                                                      INT 0087
C3                                                               INT 0088
      REM FLOATING POINT ADAMS-MOULTON, RUNGE-KUTTA INTEGRATION  INT 0089
DE2F  TRA DE2F+0293        SETUP ENTRY                           INT 0090
      SXD DE2F+0240,1            INTEGRATION ENTRY               INT 0091
      SXD DE2F+0241,2                                            INT 0092
      SXD DE2F+0242,4                                            INT 0093
      TRA DE2F+0005        SWITCH 1                              INT 0094
      CLA DE2F+0285        3 TO ACC                              INT 0095
      CAS DE2F+0228        TEST ALPHA                            INT 0096
      TRA DE2F+0186                                              INT 0097
      TRA DE2F+0175                                              INT 0098
      LXA DE2F+0229,1         Y PRIMES TO D                      INT 0099
      CLA *,1                                                    INT 0100
      STO *,1                  AND                               INT 0101
      CLA *,1                                                    INT 0102
      STO *,1                 DOUBLE PRECISION                   INT 0103
      CLA *,1                                                    INT 0104
      STO *,1                  YS TO TS2                         INT 0105
      TIX DE2F+0010,1,1           END 3F LOOP                    INT 0106
```

```
LXA DE2F+0229,1                                          INT 0107
LDQ DE2F+0223                                            INT 0108
FMP *,1                  D3 SUB I                        INT 0109
STO COMMON+000                                           INT 0110
CLA *,1                     PLANT Y SUB I                INT 0111
STO DE2F+0267          IN SFA                            INT 0112
CLA *,1                   SUBROUTINE                     INT 0113
STO DE2F+0268                                            INT 0114
LDQ DE2F+0222                                            INT 0115
FMP *,1                  D2 SUB I                        INT 0116
FAD COMMON+000                                           INT 0117
STO COMMON+000                                           INT 0118
LDQ DE2F+0221                                            INT 0119
FMP *,1                  D1 SUB I                        INT 0120
FAD COMMON+000                                           INT 0121
STO COMMON+000                                           INT 0122
LDQ DE2F+0220                                            INT 0123
FMP *,1                  D SUB I                         INT 0124
FAD COMMON+000                                           INT 0125
STO COMMON+000                                           INT 0126
LDQ *                  LOAD H                            INT 0127
FMP COMMON+000         DELTA YI UPPER P                  INT 0128
TSX DE2F+0269,2            ADD TO YI                     INT 0129
STO *,1                    STORE IN TS1                  INT 0130
TIX DE2F+0018,1,1              END OF LOOP               INT 0131
CLA *                                                    INT 0132
FAD *                  X+H                               INT 0133
STO *                                                    INT 0134
LXA DE2F+0229,1                                          INT 0135
CLA *,1                                                  INT 0136
STO *,1                                                  INT 0137
TIX DE2F+0046,1,1              END OF LOOP               INT 0138
TSX 0,4                EVALUATE DERIVATIVES              INT 0139
LXA DE2F+0229,1                                          INT 0140
LDQ DE2F+0227                                            INT 0141
FMP *,1                  D2 SUB I                        INT 0142
STO COMMON+000                                           INT 0143
CLA *,1                    YI FROM TS2                   INT 0144
STO DE2F+0267                                            INT 0145
CLA *,1                    DP EXT.                       INT 0146
STO DE2F+0268                                            INT 0147
LDQ DE2F+0226                                            INT 0148
FMP *,1                  D1 SUBI                         INT 0149
FAD COMMON+000                                           INT 0150
STO COMMON+000                                           INT 0151
LDQ DE2F+0225                                            INT 0152
FMP *,1                  D SUBI                          INT 0153
FAD COMMON+000                                           INT 0154
STO COMMON+000                                           INT 0155
LDQ DE2F+0224                                            INT 0156
FMP *,1                    Y PRIME SUBI                  INT 0157
FAD COMMON+000                                           INT 0158
STO COMMON+000                                           INT 0159
```

```
LDQ *                      LOAD H                                INT 0160
FMP COMMON+000             DELTA YI UPPER C                      INT 0161
TSX DE2F+0269,2                ADD TO Y I                        INT 0162
STO *,1                                                          INT 0163
STQ *,1                                                          INT 0164
TIX DE2F+0051,1,1                  END OF LOOP                   INT 0165
STZ DE2F+0230                                                    INT 0166
LXA DE2F+0229,1                                                  INT 0167
CLA *,1                                                          INT 0168
STO DE2F+0243                                                    INT 0169
CLA *,1                                                          INT 0170
STO DE2F+0244                                                    INT 0171
TSX DE2F+0246,2                                                  INT 0172
TIX DE2F+0078,1,1                  END OF LOOP                   INT 0173
TRA DE2F+0085              SWITCH 2                              INT 0174
CLA DE2F+0234                                                    INT 0175
CAS DE2F+0230                                                    INT 0176
TRA DE2F+0090                                                    INT 0177
TRA DE2F+0089                                                    INT 0178
TRA DE2F+0115              DECREASE H SWITCH                     INT 0179
CLA DE2F+0235                                                    INT 0180
CAS DE2F+0230                                                    INT 0181
TRA DE2F+0164              INCREASE H SWITCH                     INT 0182
TRA DE2F+0211                                                    INT 0183
TRA DE2F+0211                                                    INT 0184
LXA DE2F+0229,1                                                  INT 0185
CLA *,1                                                          INT 0186
STO *,1                        D2 TO D3                          INT 0187
CLA *,1                                                          INT 0188
STO *,1                      D1 TO D2                            INT 0189
CLA *,1                                                          INT 0190
STO *,1                      D TO D1                             INT 0191
TIX DE2F+0096,1,1                END OF LOOP                     INT 0192
CLA DE2F+0228                                                    INT 0193
ADD DE2F+0284              ALPHA PLUS ONE                        INT 0194
STO DE2F+0228                                                    INT 0195
TSX 0,4                    EVALUATE DERIVATIVES                  INT 0196
CLA DE2F+0230                                                    INT 0197
FDP DE2F+0290              E                                     INT 0198
STQ COMMON+000                                                   INT 0199
CLA COMMON+000             GET E IN ACC                          INT 0200
LXD DE2F+0240,1                                                  INT 0201
LXD DE2F+0241,2                                                  INT 0202
LXD DE2F+0242,4                                                  INT 0203
TRA 1,4                    EXIT                                  INT 0204
CLA *                                                            INT 0205
SSP                                                              INT 0206
CAS DE2F+0238              TEST H WITH HMIN                      INT 0207
TRA DE2F+0121                                                    INT 0208
TRA DE2F+0121                                                    INT 0209
TRA DE2F+0211                                                    INT 0210
LDQ *                                                            INT 0211
STQ DE2F+0233              STORE OLD H                           INT 0212
```

```
FMP DE2F+0239        BETA TIMES H                INT 0213
STO *                                            INT 0214
CLA DE2F+0285                                    INT 0215
CAS DE2F+0228        TEST ALPHA                  INT 0216
HTR DE2F+0127                                    INT 0217
TRA DE2F+0130                                    INT 0218
TRA DE2F+0144                                    INT 0219
LDQ DE2F+0289                                    INT 0220
FMP DE2F+0233        4H                          INT 0221
CHS                                              INT 0222
FAD *                X-4H                        INT 0223
STO *                                            INT 0224
LXA DE2F+0229,1                                  INT 0225
CLA *,1                                          INT 0226
STO *,1                                          INT 0227
CLA *,1                                          INT 0228
STO *,1                                          INT 0229
CLA *,1                                          INT 0230
STO *,1                                          INT 0231
TIX DE2F+0136,1,1       END OF LOOP              INT 0232
TRA DE2F+0161        JUMP TO SET ALPHA           INT 0233
CLA DE2F+0233                                    INT 0234
CHS                                              INT 0235
FAD *                X-H                         INT 0236
STO *                                            INT 0237
LXA DE2F+0229,1                                  INT 0238
CLA *,1                                          INT 0239
STO *,1                  D TO YI PRIME           INT 0240
CLA *,1                  TS2 TO YI               INT 0241
STO *,1                                          INT 0242
CLA *,1                                          INT 0243
STO *,1                                          INT 0244
TIX DE2F+0149,1,1                                INT 0245
LXA DE2F+0229,1                                  INT 0246
LDQ DE2F+0291        CONVERT DP YI               INT 0247
FMP *,1                 TO YI, QI                INT 0248
STO *,1                                          INT 0249
TIX DE2F+0157,1,1          END OF LOOP           INT 0250
STZ DE2F+0228                                    INT 0251
STZ DE2F+0231                                    INT 0252
TRA DE2F+0107                                    INT 0253
CLA *                                            INT 0254
SSP                                              INT 0255
CAS DE2F+0237        TEST H WITH HMAX            INT 0256
TRA DE2F+0095        GO TO SHIFT DS              INT 0257
TRA DE2F+0213                                    INT 0258
TRA DE2F+0213                                    INT 0259
CLA *                                            INT 0260
FDP DE2F+0239        H DIVIDED BY BETA           INT 0261
STO *                                            INT 0262
TSX 0,4              EVALUATE DERIVATIVES        INT 0263
TRA DE2F+0156                                    INT 0264
LXA DE2F+0229,1                                  INT 0265
```

```
CLA *,1                                                     INT 0266
FDP DE2F+0291        QI DIVIDED BY -3                       INT 0267
CLA *,1                                                     INT 0268
STQ COMMON+000                                              INT 0269
FAD COMMON+000                                              INT 0270
STO *,1                                                     INT 0271
STQ *,1                                                     INT 0272
TIX DE2F+0176,1,1                                           INT 0273
TRA DE2F+0009                                               INT 0274
STO *,1                   DUMMY ORDER                       INT 0275
LDQ DE2F+0229                                               INT 0276
MPY DE2F+0228                                               INT 0277
STQ COMMON+000                                              INT 0278
CLA COMMON+000                                              INT 0279
ADD DE2F+0185                                               INT 0280
STO DE2F+0194                                               INT 0281
LXA DE2F+0229,1                                             INT 0282
CLA *,1                                                     INT 0283
STO 0                STORE DERIVATIVES                      INT 0284
TIX DE2F+0193,1,1                                           INT 0285
CLA DE2F+0228                                               INT 0286
TNZ DE2F+0204                                               INT 0287
LXA DE2F+0229,1                                             INT 0288
CLA *,1                                                     INT 0289
STO *,1                                                     INT 0290
CLA *,1                                                     INT 0291
STO *,1                                                     INT 0292
TIX DE2F+0199,1,1                                           INT 0293
CLA DE2F+0228                                               INT 0294
ADD DE2F+0284        ALPHA PLUS ONE                         INT 0295
STO DE2F+0228                                               INT 0296
TSX DE2F+0454,4        RUNGA-KUTTA ENTRY                    INT 0297
STZ COMMON+000                                              INT 0298
CLA COMMON+000       ZERO TO ACC                            INT 0299
TRA DE2F+0111                                               INT 0300
STZ DE2F+0231                                               INT 0301
TRA DE2F+0095                                               INT 0302
CLA DE2F+0231                                               INT 0303
ADD DE2F+0284        R+1                                    INT 0304
STO DE2F+0231                                               INT 0305
CAS DE2F+0292                                               INT 0306
HTR DE2F+0217                                               INT 0307
TRA DE2F+0170                                               INT 0308
TRA DE2F+0095                                               INT 0309
DEC 2.291666667                                             INT 0310
DEC -2.458333333                                            INT 0311
DEC 1.541666667                                             INT 0312
DEC -3.75E-1                                                INT 0313
DEC 3.75E-1                                                 INT 0314
DEC 7.916666667E-1                                          INT 0315
DEC -2.083333333E-1                                         INT 0316
DEC 4.166666667E-2                                          INT 0317
BSS 4                                                       INT 0318
```

```
BSS 8                                                   INT 0319
BSS 3                                                   INT 0320
HTR                     HI UPPER C                      INT 0321
HTR                     YI UPPER P                      INT 0322
HTR                     DIVISOR                         INT 0323
CLA DE2F+0243           ENTRY                           INT 0324
SSP                                                     INT 0325
CAS DE2F+0236           TEST FOR DIVISOR                INT 0326
TRA DE2F+0254                                           INT 0327
TRA DE2F+0254                                           INT 0328
CLA DE2F+0236                                           INT 0329
STO DE2F+0245                                           INT 0330
TRA DE2F+0255                                           INT 0331
STO DE2F+0245                                           INT 0332
CLA DE2F+0244                                           INT 0333
FSB DE2F+0243                                           INT 0334
SSP                                                     INT 0335
FDP DE2F+0245                                           INT 0336
STQ COMMON+000                                          INT 0337
CLA DE2F+0230                                           INT 0338
CAS COMMON+000                                          INT 0339
TRA 1,2                                                 INT 0340
TRA 1,2                                                 INT 0341
CLA COMMON+000                                          INT 0342
STO DE2F+0230                                           INT 0343
TRA 1,2                                                 INT 0344
HTR                     A1                              INT 0345
HTR                     A2                              INT 0346
UFA DE2F+0267           ENTRY                           INT 0347
STO DE2F+0267                                           INT 0348
STQ COMMON+000          SPECIAL                         INT 0349
CLA COMMON+000          FLOATING                        INT 0350
UFA DE2F+0268           ADDITION                        INT 0351
FAD DE2F+0267           SUBROUTINE                      INT 0352
TRA 1,2                                                 INT 0353
TRA DE2F+0005           SWITCH 1, A LEG                 INT 0354
TRA DE2F+0207           SWITCH 1, B LEG                 INT 0355
TRA DE2F+0085           SWITCH 2, A LEG                 INT 0356
TRA DE2F+0095           SWITCH 2, B LEG                 INT 0357
TRA DE2F+0115           DECREASE H SWITCH, TEST LEG     INT 0358
TRA DE2F+0121           DECREASE H SWITCH, NO TEST LEG  INT 0359
TRA DE2F+0164           INCREASE H SWITCH, TEST LEG     INT 0360
TRA DE2F+0213           INCREASE H SWITCH, NO TEST LEG  INT 0361
DEC 1                                                   INT 0362
DEC 3                                                   INT 0363
DEC 1.                                                  INT 0364
DEC 5E-1                ONE HALF                        INT 0365
DEC 100.                                                INT 0366
DEC 4.                                                  INT 0367
DEC 14.                                                 INT 0368
DEC -3.                                                 INT 0369
DEC 3                   SPECIAL TEST NO. FOR R          INT 0370
SXD DE2F+0241,2                                         INT 0371
```

```
SXD DE2F+0242,4                                          INT 0372
CLA DE2F+0276                                            INT 0373
STO DE2F+0004        SET                                 INT 0374
CLA DE2F+0278        SWITCHES                            INT 0375
STO DE2F+0084        TO                                  INT 0376
CLA DE2F+0280        NORMAL                              INT 0377
STO DE2F+0089        POSITIONS                           INT 0378
CLA DE2F+0282                                            INT 0379
STO DE2F+0092                                            INT 0380
CLA 2,4                                                  INT 0381
PAX 0,2                                                  INT 0382
TRA DE2F+0308,2              SELECT OPTION               INT 0383
TRA DE2F+0346        OPTION 2                            INT 0384
TRA DE2F+0343        OPTION 1                            INT 0385
LDQ 3,4              OPTION 0                            INT 0386
FMP DE2F+0290        14E (UPPER)                         INT 0387
STO DE2F+0234                                            INT 0388
CLA 8,4                                                  INT 0389
TNZ DE2F+0314        TEST BETA                           INT 0390
CLA DE2F+0287                                            INT 0391
STO DE2F+0239        STORE BETA                          INT 0392
CLA 4,4                                                  INT 0393
TNZ DE2F+0318                                            INT 0394
CLA DE2F+0288                                            INT 0395
STO DE2F+0235        STORE M                             INT 0396
CLA DE2F+0234                                            INT 0397
FDP DE2F+0235                                            INT 0398
STQ DE2F+0235        STORE 14E (LOWER)                   INT 0399
CLA 5,4                                                  INT 0400
TNZ DE2F+0325        TEST A                              INT 0401
CLA DE2F+0286                                            INT 0402
STO DE2F+0236        STORE A                             INT 0403
CLA 6,4                                                  INT 0404
TNZ DE2F+0331        TEST.H MAX                          INT 0405
CLA DE2F+0283                                            INT 0406
STO DE2F+0092                                            INT 0407
TRA DE2F+0335                                            INT 0408
STO DE2F+0237                                            INT 0409
LDQ DE2F+0239                                            INT 0410
FMP DE2F+0237                                            INT 0411
STO DE2F+0237        STORE BETA (H MAX)                  INT 0412
CLA 7,4                                                  INT 0413
TNZ DE2F+0340        TEST H MIN                          INT 0414
CLA DE2F+0281                                            INT 0415
STO DE2F+0089                                            INT 0416
TRA DE2F+0348                                            INT 0417
FDP DE2F+0239                                            INT 0418
STQ DE2F+0238                                            INT 0419
TRA DE2F+0348                                            INT 0420
CLA DE2F+0277                                            INT 0421
STO DE2F+0004                                            INT 0422
TRA DE2F+0348                                            INT 0423
CLA DE2F+0279                                            INT 0424
```

```
STO DE2F+0084                                      INT 0425
CLA 1,4                                            INT 0426
STO DE2F+0232                                      INT 0427
STO DE2F+0352                                      INT 0428
TSX DE2F+0453,4       SETUP RK SUBROUTINE          INT 0429
PZE                PARAMETER WORD                  INT 0430
STZ DE2F+0228      SET ALPHA TO ZERO               INT 0431
STZ DE2F+0231      R 0                             INT 0432
CLA DE2F+0232                                      INT 0433
STA DE2F+0357                                      INT 0434
CLA 0                                              INT 0435
STO DE2F+0229      STORE N                         INT 0436
CLA DE2F+0232                                      INT 0437
ARS 18                                             INT 0438
STA DE2F+0049      SETUP                           INT 0439
STA DE2F+0106      DERIVATIVE                      INT 0440
STA DE2F+0173      EVALUATIONS                     INT 0441
CLA DE2F+0232                                      INT 0442
ADD DE2F+0284      T=1                             INT 0443
STA DE2F+0042                                      INT 0444
STA DE2F+0044                                      INT 0445
STA DE2F+0133                                      INT 0446
STA DE2F+0134                                      INT 0447
STA DE2F+0146                                      INT 0448
STA DE2F+0147                                      INT 0449
ADD DE2F+0284      T=2                             INT 0450
STA DE2F+0037                                      INT 0451
STA DE2F+0043                                      INT 0452
STA DE2F+0070                                      INT 0453
STA DE2F+0115                                      INT 0454
STA DE2F+0121                                      INT 0455
STA DE2F+0124                                      INT 0456
STA DE2F+0164                                      INT 0457
STA DE2F+0170                                      INT 0458
STA DE2F+0172                                      INT 0459
ADD DE2F+0284      T=3 EQUALS D                    INT 0460
ADD DE2F+0229      D=N                             INT 0461
STA DE2F+0010                                      INT 0462
STA DE2F+0021                                      INT 0463
STA DE2F+0047                                      INT 0464
STA DE2F+0073                                      INT 0465
STA DE2F+0078                                      INT 0466
STA DE2F+0139                                      INT 0467
STA DE2F+0152                                      INT 0468
STA DE2F+0178                                      INT 0469
STA DE2F+0181                                      INT 0470
STA DE2F+0199                                      INT 0471
ADD DE2F+0229      D+2N                            INT 0472
STA DE2F+0012                                      INT 0473
STA DE2F+0067                                      INT 0474
STA DE2F+0137                                      INT 0475
STA DE2F+0150                                      INT 0476
STA DE2F+0193                                      INT 0477
```

```
ADD DE2F+0229          D+3N                              INT 0478
STA DE2F+0014                                            INT 0479
STA DE2F+0023                                            INT 0480
STA DE2F+0056                                            INT 0481
STA DE2F+0074                                            INT 0482
STA DE2F+0141                                            INT 0483
STA DE2F+0154                                            INT 0484
STA DE2F+0158                                            INT 0485
STA DE2F+0159                                            INT 0486
STA DE2F+0176                                            INT 0487
STA DE2F+0182                                            INT 0488
STA DE2F+0201                                            INT 0489
ADD DE2F+0229          D+4N                              INT 0490
STA DE2F+0040                                            INT 0491
STA DE2F+0046                                            INT 0492
STA DE2F+0080                                            INT 0493
ADD DE2F+0229          D+5N                              INT 0494
STA DE2F+0011                                            INT 0495
STA DE2F+0054                                            INT 0496
STA DE2F+0151                                            INT 0497
ADD DE2F+0229          D+6N                              INT 0498
STA DE2F+0015                                            INT 0499
STA DE2F+0153                                            INT 0500
ADD DE2F+0229          D+7N                              INT 0501
STA DE2F+0138                                            INT 0502
STA DE2F+0200                                            INT 0503
ADD DE2F+0229          D+8N                              INT 0504
STA DE2F+0140                                            INT 0505
STA DE2F+0202                                            INT 0506
ADD DE2F+0229          D+9N                              INT 0507
STA DE2F+0019                                            INT 0508
STA DE2F+0097                                            INT 0509
STA DE2F+0136                                            INT 0510
STA DE2F+0185                                            INT 0511
ADD DE2F+0229          D+10N                             INT 0512
STA DE2F+0026                                            INT 0513
STA DE2F+0052                                            INT 0514
STA DE2F+0096                                            INT 0515
STA DE2F+0099                                            INT 0516
ADD DE2F+0229          D+11N                             INT 0517
STA DE2F+0030                                            INT 0518
STA DE2F+0059                                            INT 0519
STA DE2F+0098                                            INT 0520
STA DE2F+0101                                            INT 0521
ADD DE2F+0229          D+12N                             INT 0522
STA DE2F+0013                                            INT 0523
STA DE2F+0034                                            INT 0524
STA DE2F+0063                                            INT 0525
STA DE2F+0100                                            INT 0526
STA DE2F+0149                                            INT 0527
LXD DE2F+0241,2                                          INT 0528
LXD DE2F+0242,4                                          INT 0529
TRA 9,4                EXIT                              INT 0530
```

```
TRA DE2F+0562            TO SETUP REGION              INT 0531
SXD DE2F+0553,1             SAVE INDEX REGISTERS      INT 0532
SXD DE2F+0554,2          FROM                         INT 0533
SXD DE2F+0555,4          MAIN PROGRAM                 INT 0534
CLA                                                   INT 0535
FDP DE2F+0520            CALCULATE                    INT 0536
STQ DE2F+0525            H DIVIDED BY 2               INT 0537
STQ DE2F+0541                                         INT 0538
LXA DE2F+0551,2               SET PARAMETER INDEX     INT 0539
LXA DE2F+0556,1          SET N INDEX.                 INT 0540
LDQ ,1                                                INT 0541
FMP DE2F+0551,2                                       INT 0542
STO ,1                                                INT 0543
LDQ                      CALCULATE NEW                INT 0544
FMP ,1                   VALUE OF P                   INT 0545
FAD ,1                                                INT 0546
STO ,1                                                INT 0547
FDP DE2F+0552,2                                       INT 0548
STQ COMMON+000                                        INT 0549
LDQ ,1                   CALCULATE NEW                INT 0550
FMP DE2F+0553,2              VALUE OF R               INT 0551
FAD COMMON+000                                        INT 0552
STO COMMON+000                                        INT 0553
TZE DE2F+0492                                         INT 0554
ARS 27                                                INT 0555
STO COMMON+001           TEST VALUES                  INT 0556
CLA ,1                                                INT 0557
TZE DE2F+0492            TO                           INT 0558
SSP                                                   INT 0559
ARS 27                   DETERMINE                    INT 0560
SBM COMMON+001                                        INT 0561
TMI DE2F+0492            IF SHIFTING                  INT 0562
TZE DE2F+0492                                         INT 0563
STA DE2F+0489            IS NECESSARY                 INT 0564
STA DE2F+0490                                         INT 0565
CLA COMMON+000                                        INT 0566
ARS                                                   INT 0567
ALS                                                   INT 0568
STO COMMON+000                                        INT 0569
CLA ,1                   CALCULATE NEW                INT 0570
FAD COMMON+000           VALUE OF Z                   INT 0571
STO ,1                                                INT 0572
TRA DE2F+0558,2                                       INT 0573
LDQ ,1                                                INT 0574
FMP DE2F+0556,2               CALCULATE               INT 0575
STO ,1                                                INT 0576
LDQ ,1                   NEW VALUE                    INT 0577
FMP DE2F+0555,2                                       INT 0578
FAD ,1                                                INT 0579
STO ,1                   OF Q                         INT 0580
LDQ COMMON+000                                        INT 0581
FMP DE2F+0554,2                                       INT 0582
FAD ,1                                                INT 0583
```

```
STO ,1                                                    INT 0584
TIX DE2F+0463,1,1          TEST N                         INT 0585
CLA                                                       INT 0586
FAD DE2F+0557,2            INCREASE X                     INT 0587
STO                                                       INT 0588
SXD DE2F+0557,2                                           INT 0589
TSX ,4                 FIND DERIVITIVES                   INT 0590
LXD DE2F+0557,2                                           INT 0591
TIX DE2F+0462,2,8          TEST PASS NO.                  INT 0592
LXD DE2F+0553,1             RESTORE                       INT 0593
LXD DE2F+0554,2            INDEX                          INT 0594
LXD DE2F+0555,4            REGISTERS                      INT 0595
TRA 1,4                                                   INT 0596
DEC 0                                                     INT 0597
DEC 2.                                                    INT 0598
DEC -1.                                                   INT 0599
DEC 3.                                                    INT 0600
DEC -.5                                                   INT 0601
DEC 1.                                                    INT 0602
DEC                                                       INT 0603
TRA DE2F+0496                                             INT 0604
DEC 0                                                     INT 0605
DEC 2.                                                    INT 0606
DEC -.5                                                   INT 0607
DEC -1.                                                   INT 0608
DEC .5                                                    INT 0609
DEC -3.                                                   INT 0610
DEC 0                                                     INT 0611
TRA DE2F+0558                                             INT 0612
DEC -.5                                                   INT 0613
DEC 1.                                                    INT 0614
DEC 0                                                     INT 0615
DEC -1.                                                   INT 0616
DEC 0                                                     INT 0617
DEC 1.                                                    INT 0618
DEC                                                       INT 0619
TRA DE2F+0496                                             INT 0620
DEC 2.                                                    INT 0621
DEC 6.                                                    INT 0622
DEC 1.                                                    INT 0623
DEC 3.                                                    INT 0624
DEC -.5                                                   INT 0625
DEC -3.                                                   INT 0626
DEC 0                                                     INT 0627
TRA DE2F+0496                                             INT 0628
PZE 32,,                                                  INT 0629
PZE 1,,                                                   INT 0630
BSS 5                                                     INT 0631
CLA ,1                                                    INT 0632
FDP DE2F+0556,2                                           INT 0633
STQ ,1                                                    INT 0634
TRA DE2F+0499                                             INT 0635
SXD DE2F+0553,1          SAVE INDEX REGISTERS             INT 0636
```

```
       SXD DE2F+0555,4          FROM MAIN PROGRAM            INT 0637
       CLA 1,4                                               INT 0638
       STA DE2F+0553                                         INT 0639
       STA DE2F+0570                                         INT 0640
       ARS 18                                                INT 0641
       STA DE2F+0512           SET ADDRESS OF                INT 0642
       STA DE2F+0603           DERIVITIVE ROUTINE            INT 0643
       CLA                     STORE VALUE                   INT 0644
       STO DE2F+0556           OF N                          INT 0645
       CLA DE2F+0553                                         INT 0646
       ADD DE2F+0552                                         INT 0647
       STA DE2F+0508           STORE ADDRESS                 INT 0648
       STA DE2F+0510           OF X                          INT 0649
       ADD DE2F+0552                                         INT 0650
       STA DE2F+0466           STORE ADDRESS                 INT 0651
       STA DE2F+0457           OF H                          INT 0652
       ADD DE2F+0552                                         INT 0653
       ADD DE2F+0556                                         INT 0654
       STA DE2F+0479           STORE ADDRESS                 INT 0655
       STA DE2F+0492           OF Y                          INT 0656
       STA DE2F+0494                                         INT 0657
       ADD DE2F+0556           STORE ADDRESS                 INT 0658
       STA DE2F+0467           OF DERIVITIVE                 INT 0659
       ADD DE2F+0556                                         INT 0660
       STA DE2F+0605                                         INT 0661
       STA DE2F+0560           STORE                         INT 0662
       STA DE2F+0558                                         INT 0663
       STA DE2F+0472           ADDRESS                       INT 0664
       STA DE2F+0496                                         INT 0665
       STA DE2F+0498           OF                            INT 0666
       STA DE2F+0501                                         INT 0667
       STA DE2F+0502           Q                             INT 0668
       STA DE2F+0505                                         INT 0669
       STA DE2F+0506                                         INT 0670
       ADD DE2F+0556                                         INT 0671
       STA DE2F+0463                                         INT 0672
       STA DE2F+0465           STORE ADDRESS                 INT 0673
       STA DE2F+0468                                         INT 0674
       STA DE2F+0469           OF P                          INT 0675
       STA DE2F+0499                                         INT 0676
       TSX ,4                  FIND.INITIAL DERIVITIVES      INT 0677
       LXA DE2F+0556,1                                       INT 0678
       STZ ,1                  SET ORGINAL Q                 INT 0679
       TIX DE2F+0605,1,1          TO ZERO                    INT 0680
       LXD DE2F+0553,1              RESTORE INDEX            INT 0681
       LXD DE2F+0555,4            REGISTERS                  INT 0682
       TRA 2,4                                               INT 0683
COMMON BSS 2                                                 INT 0684
 R     END                                                   INT 0685
```

H. SUBROUTINE RINDEX

This subroutine permits the calculation of the refractive index, its spatial derivatives, and the absorption coefficient, all as functions of the local atmosphere and its state of ionization. This subroutine also permits the output of "debugging data" which is called the R vector whose components are defined in Table 3. See subroutine OUTPUT for additional information.

$R(1) = X$

$R(2) = PXR = \frac{\partial X}{\partial r}$

$R(3) = PXTHET = \frac{\partial X}{\partial \theta}$

$R(4) = PXPHI = \frac{\partial X}{\partial \varphi}$

$R(5) = Y$

$R(6) = YR = Y_r$

$R(7) = YTHETA = Y_\theta$

$R(8) = YPHI = Y_\varphi$

$R(9) = PYR = \partial Y / \partial r$

$R(10) = PYTHET = \partial Y / \partial \theta$

$R(11) = PYPHI = \partial Y / \partial \varphi$

$R(12) = Z$

$R(13) = PZR = \partial Z / \partial r$

$R(14) = PZTHET = \partial Z / \partial \theta$

$R(15) = PZPHI = \partial Z / \partial \varphi$

$R(16) = COSPSI = \cos \psi$

$R(17) = SINPSI = \sin \psi$

$R(18) = YSI = Y \sin \psi$

$R(19) = YCI = Y \cos \psi$

$R(20) = TE1 = (1 - X)^2 - Z^2$

Table 3. Nomenclature Describing the R Vector.
(Page 1 of 4)

$R(21) = TE2 = (YSI)^4 + 4TE1(YCI)^2$

$R(22) = TE3 = 8(YCI)^2\ Z(1 - X)$

$R(23) = R2S = R_S{}^2$

$R(24) = R1S = R_S$

$R(25) = THET2S = 2\theta_S$

$R(26) = THET1s = \theta_S$

$R(27) = S1 = S_1$

$R(28) = S2 = S_2$

$R(29) = D1 = d_1$

$R(30) = D2 = d_2$

$R(31) = TE4 = d_1{}^2 + d_2{}^2$

$R(32) = TE5 = 2X\left[Zd_1 + (1 - X)d_2\right]/TE4$

$R(33) = TE6 = 1 - \left[2X(1 - X)d_1 - Zd_2\right]/TE4$

$R(34) = R2M = R_M{}^2$

$R(35) = R1M = R_M$

$R(36) = THET2M = 2\theta_M$

$R(37) = THET1M = \theta_M$

$R(38) = AM1 = M_1$

$R(39) = AM2 = M_2$

$R(40) = TE7 = M_1 d_1 - M_2 d_2$

Table 3. Nomenclature Describing the R Vector.
(Page 2 of 4)

$R(41) = TE8 = M_1 d_2 + M_2 d_1$

$R(42) = AO = a_o$

$R(43) = BO = b_o$

$R(44) = TE9 = S_1^2 + S_2^2$

$R(45) = A4 = a_4$

$R(46) = B4 = b_4$

$R(47) = A5 = a_5$

$R(48) = B5 = b_5$

$R(49) = PNPX = \partial\mu/\partial X$

$R(50) = TE10 = (\sin\psi)^2(YSI)^2 + 2TE1\cos^2\psi$

$R(51) = TE11 = 4Z(1 - X)\cos^2\psi$

$R(52) = A6 = a_6$

$R(53) = B6 = b_6$

$R(54) = PNPY = \partial\mu/\partial Y$

$R(55) = A7 = a_7$

$R(56) = B7 = b_7$

$R(57) = PNPZ = \partial\mu/\partial Z$

$R(58) = A8 = a_8$

$R(59) = B8 = b_8$

$R(60) = TE13 = 1/(2\cos^2\theta + \frac{1}{2}\sin^2\theta)$

Table 3. Nomenclature Describing the R Vector.
(Page 3 of 4)

$R(61) = A1 = a_1$

$R(62) = B1 = b_1$

$R(63) = A2 = a_2$

$R(64) = B2 = b_2$

$R(65) = TE12 = (W_1)^2\ YSI$

$R(66) = PPSIPT = \partial\psi/\partial\theta$

$R(67) = PPSIPR = \partial\psi/\partial r$

$R(68) = PPSIPP = \partial\psi/\partial\varphi$

$R(69) = TE14 = \sqrt{\sigma_r{}^2 + \sigma_\theta{}^2 + \sigma_\varphi{}^2}$

Table 3. Nomenclature Describing the R Vector.
(Page 4 of 4)

```
C     RINDEX WITH MAGNETIC FIELD  DECEMBER 22,1960   IBM-7090
1     SUBROUTINE  RINDEX
      COMMON RECORD,V,W,M
      DIMENSION  RECORD(12),V(11),W(250),R(75)
      PI = 3.1415927
4     CALL ELECTX (X,PXR,PXTHET,PXPHI)
      R(1) = X
      R(2) =   PXR
      R(3)= PXTHET
      R(4)=PXPHI
      SIGN = W(67)
5     CALL MAGY (Y,YR,YTHETA,YPHI,PYR ,PYTHET,PYPHI )
      R(5)=  Y
      R(6)=YR
      R(7)  = YTHETA
      R(8) = YPHI
      R(9) = PYR
      R(10) =  PYTHET
      R(11) = PYPHI
6     CALL COLFRZ( Z,PZR,PZTHET,PZPHI)
      R(12) =  Z
      R(13) = PZR
      R(14) = PZTHET
      R(15) =PZPHI
      W(63) = X
      W(65) = Y
      W(64) = Z
7     COSPSI=(V(7)*YR + V(8)*YTHETA +V(9)* YPHI)/(Y* W(1))
      R(16) = COSPSI
8     SINPSI = SQRTF( 1.0 - COSPSI*COSPSI )
      R(17) = SINPSI
      YSI =  Y*SINPSI
      R(18) = YSI
      YCI =  Y*COSPSI
      R(19) = YCI
      IF(Z-1.0E-18) 9,10,10
9     T  = 0.0
      G  TO
10    T1 = Z*Z
11    TE1 = ((1.0- X)*(1.0-X) -   )
      R(20) = TE1
      TEE =  YSI*YSI
      TEA =  YCI*YCI
12    TE2 = TEE*TEE + 4.0*TEA*TE1
      R(21) = TE2
      TEB = Z*(1.0-X)
13    TE3 = 8.0*TEA*TEB
      R(22) = TE3
14    R4S = TE2*TE2 + TE3*TE3
      F2S  =  SQRTF(R4S)
      R(23) = F2
15    R1S =  SQRTF (R2S
16    THET2S =   QATAN(TE2, -TE3)
      IF(PI - THET2S)200,200,17
200   R1S = -R1S
17    THET1S = 0.5*THET2S
      R(24) = R1S
      R(25) = THET2S
```

```
      R(26) = THET1S
18    S1 =  R1S* COSF(THET1S)*SIGN
      R(27) = S1
19    S2 =  R1S* SINF(THET1S)*SIGN
      R(28) = S2
20    D1 = 2.0*(1.0-X-Z*Z) - TEE + S1
      R(29) = D1
21    D2 =  S2 + Z*(X-2.0)*2.0
      R(30) = D2
22    TE4 =  D1*D1 + D2*D2
      R(31) = TE4
23    TE5 =  2.0*X*(Z*D1+(1.0-X)*D2) /TE4
      R(32) = TE5
      TEC = (1.0-X)*D1
24    TE6 = 1.0 - (2.0*X*(TEC-Z*D2) /TE4)
      R(33) = TE6
25    R4M = TE6*TE6 + TE5*TE5
      R2M  =  SQRTF(R4M)
      R(34) = R2M
26    R1M =    SQRTF(R2M)
27    THET2M =   ATAN (TE6, TE5)
      IF(PI - THET2M)201,201,28
201   R1M  = - R1M
28    THET1M = 0.5 * THET2M
      R(35) = R1M
      R(36) = THET2M
      R(37) = THET1M
29    AM1=  R1M *COSF(THET1M)
      R(38) = AM1
30    AM2=  R1M *SINF (THET1M)
      R(39) = AM2
31    W(1) = AM1
32    W(2) = -AM2
33    TE7 = AM1*D1 - AM2*D2
      R(40) = TE7
34    TE8 = AM1*D2 + AM2*D1
      R(41) = TE8
      TED = TE7*TE7
      TEF = TE8*TE8
35    A0 =  TE7/(TED +TEF)
      R(42) = A0
36    B0  = TE8*A0/TE7
      R(43) = B0
37    TE9 = S1*S1 +S2*S2
      R(44) = TE9
      Q5  =  2.0*TEA
      TEG = (1.0-X)*S1 - Z*S2
38    A4 = 1.0 + Q5*TEG/TE9
      R(45) = A4
      TEH =(Z*S1 + (1.0-X)*S2)/TE9
39    B4  = Z + TEH*Q5
      R(46) =  B4
40    A5  = 2.0*X*(TEC-Z*D2)/TE4
      R(47) = A5
41    B5 =  2.0*X*(Z*D1 +(1.0-X)*D2) /TE4
      R(48) = B5
      TEJ = 2.0*X-1.0 - A4*A5 + B4*B5
      TEK = Z + B4*A5 + A4*B5
```

```
42   PNPX = A0*TEJ + B0*TEK
     R(49) = PNPX
     Q5 = SINPSI**2
     Q6 =(COSPSI**2)*2.0
43   TE10 =  TEE*Q5 + Q6*TE1
     R(50) = TE10
     TEB2 = 2.0*TEB
44   TE11 =  TEE2*Q6
     R(51) = TE11
45   A6 = (TE10*S1 - TE11*S2)/ TE9
     R(52) = A6
46   B6 = (TE10*S2 + TE11*S1)/ TE9
     R(53) = B6
     Q1 = B0*A5 + B5*A0
     Q2 = A0*A5 - B0*B5
     Q3 = A6 - SINPSI*SINPSI
     Q4 = Q2*Q3
47   PNPY = Y*Q4 - Q1*B6*Y
     R(54) = PNPY
     Q5 = 2.0*TEA
48   A7 = ((2.0-X) + Q5*TEG/TE9)
     R(55) = A7
49   B7 = 2.0*(Z + TEH*TEA)
     R(56) = B7
     Q1 = X - A5*A7 + B5*B7
     Q2 = A5*B7 + B5*A7
50   PNPZ = B0*Q1 - A0*Q2
     R(57) = PNPZ
     Q3 = 1.0 - X*X
     Q4 = S1*Q3 - S2*Z*(1.0-X)
51   A8 = 2.0*(1.0 + TEA*Q4/TE9)
     R(58) = A8
     Q6 = S2*Q3 + Z*S1*(1.0+X)
52   B8 = Z*(2.0+X)+ Q5*Q6/TE9
     R(59) = B8
     Q1 = 2.0*X*X - A5*A8 + B5*B8
     Q1 = 2.0*QW(X,X) - QW(A5,A8) + QW(B5,B8)
     Q2 = X*Z + A5*B8 + B5*A8
53   W(11) = -(A0*Q1 + B0*Q2)/W(3)
      Q1 = COSF(V(5))
      Q2 = SINF(V(5))
     Q3 = 2.0*Q1*Q1 + 0.5*Q2*Q2
54   TE13 =  1.0/Q3
     R(60) = TE13
55   IF (ABSF(Y51)-1.0E-16)100,100,60
60   A1 =  2.0* TE1 -  TEE
     R(61) = A1
61   B1 =  4.0*   TEB
     R(62) = B1
62   A2 =  (1.0 +(A1*S1 -B1*S2)/TE9 )
     R(63) = A2
63   B2 =  (S1*B1 + A1*S2)/TE9
     R(64) = B2
     TAA = A2*A5
     TAB = B2*B5
     TAC = A2*B5
     TAD = B2*A5
     TAE = A0*(TAA-TAB)
```

```
      TAF = B0*(TAC+TAD)
      Q1  = YCI*YSI
64    W(10) = - Q1*(TAE-TAF)
      TAG  =  W(1)*W(1)
65    TE12 = TAG*YSI
      R(65) = TE12
      X1 = V(7)*VCI
      X2 = W(1)*YR
      X3 = X1 - X2
66    W(4) = W(10)*X3/TE12
      X1 = V(8)*VCI
      X2 = W(1)*YTHETA
      X3 = X1 - X2
67    W(5) = W(10)*X3/TE12
      X1  = V(9)*VCI
      X2  = W(1)*YPHI
      X3 = X1 - X2
68    W(6) = W(10)*X3/TE12
      X1 = V(7)*YTHETA - V(8)*YR
      X2 =  W(1)*YSI
      X3 =  X1 - X2
69    PPSIPT = X3*TE13
      R(66) = PPSIPT
70    PPSIPR = 0.0
71    PPSIPP = 0.0
72    W(7) = PNPX*PXR + PNPY*PYR + PNPZ*PZR + W(10)*PPSIPR
73    W(8) = PNPX*PXTHET + PNPY*PYTHET + PNPZ*PZTHET + W(10)*PPSIPT
74    W(9) = PNPX*PXPHI  + PNPY*PYPHI + PNPZ*PZPHI + W(10)*PPSIPP
75    GO TO 104
100   W(4)= 0.0
101   W(5)= 0.0
102   W(6)= 0.0
103   PPSIPT = TE13
      GO TO 70
104   X1  =  V(7)*V(7)
      X2  =  V(8)*V(8)
      X3  =  V(9)*V(9)
      TE14 = SQRTF( X1 + X2 + X3 )
      R(67) = PPSIPR
      R(68) = PPSIPP
      R(69) = TE14
105   V(7) =  V(7)*W(1)/TE14
106   V(8) =  V(8)*W(1)/TE14
107   V(9) =  V(9)*W(1)/TE14
      IF (W(68)) 125, 129, 126
125    IF (W(1)- W(69)) 127, 108,108
126    IF (V(4) - W(68)) 108, 127, 127
131    IF(W(70)-WTST)108,132,132
132   WTST = WTST + W(63)
127   WRITE OUTPUT TAPE 6,128,(R(I),I=1,69)
128   FORMAT(1H0,1P8E14.6)
      GO TO 108
129   IF(W(70)-1.0)130,131,131
130    WTST = W(63)
      GO TO 127
108   RETURN
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
N	32188	73674	RECORD	32561	77461	V	32549	77445	W	32438	77266			

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	1	[illegible]	129	[illegible]	00000	4	7	00023	4	8	00023	[illegible]	14	[illegible]
5	15	00042	[illegible]	[illegible]	00070	6	24	00070	7	32	00113	[illegible]	[illegible]	[illegible]
9	41	00162	[illegible]	[illegible]	[illegible]	11	44	00170	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	53	00237	15	[illegible]	[illegible]	16	57	00256	13	58	00260	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	19	[illegible]	00317	20	[illegible]	00327	[illegible]	[illegible]	[illegible]
22	73	00357	23	[illegible]	[illegible]	24	[illegible]	00415	25	[illegible]	00433	[illegible]	[illegible]	[illegible]
27	84	00452	[illegible]	[illegible]	[illegible]	201	[illegible]	00432	29	[illegible]	00434	[illegible]	[illegible]	[illegible]
30	94	[illegible]	[illegible]	[illegible]	00515	32	97	00517	33	[illegible]	00521	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	36	[illegible]	[illegible]	37	106	00567	38	[illegible]	00614	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	42	123	00723	43	127	00744	44	[illegible]	[illegible]
45	132	00765	[illegible]	[illegible]	[illegible]	47	140	01037	48	143	01055	[illegible]	145	[illegible]
50	[illegible]	01120	51	[illegible]	01151	52	[illegible]	01173	53	164	01264	54	166	[illegible]
55	[illegible]	01224	[illegible]	[illegible]	[illegible]	61	[illegible]	01340	[illegible]	[illegible]	01345	63	[illegible]	[illegible]
[illegible]	[illegible]	01427	[illegible]	[illegible]	[illegible]	66	[illegible]	01457	[illegible]	[illegible]	01474	[illegible]	[illegible]	[illegible]
69	[illegible]	01532	[illegible]	[illegible]	[illegible]	71	209	01541	[illegible]	[illegible]	01543	[illegible]	[illegible]	[illegible]
[illegible]	211	[illegible]	[illegible]	[illegible]	[illegible]	100	213	01621	101	214	01623	[illegible]	[illegible]	[illegible]
103	[illegible]	01627	[illegible]	[illegible]	[illegible]	105	225	01656	106	226	01662	107	[illegible]	[illegible]
125	[illegible]	01675	[illegible]	[illegible]	[illegible]	131	231	01707	132	232	01714	127	233	[illegible]
127	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	130	240	01735	[illegible]	242	01740			

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
1197	02255	32187	73673

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
[illegible]	0	00000	COS	5	00005	[illegible]	2	00002	MAG	1	00001	[illegible]	4	00004
QW	7	00007	SIN	6	00006	SQRT	3	00003	[illegible]	9	00011	[illegible]	8	00010

STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
R	1196	02254												

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
D2	1121	02141	D1	1120	02140	COSPSI	1119	02137	B8	1118	02136	B7	1117	[illegible]
B6	1116	02134	B5	1115	02133	B4	1114	02132	B2	1113	02131	B1	1112	[illegible]
B0	1111	02127	AM2	1110	02126	AM1	1109	02125	A8	1108	02124	A7	1107	[illegible]
A6	1106	02122	A5	1105	02121	A4	1104	02120	A2	1103	02117	A1	1102	[illegible]
A0	1101	02115	PI	1100	02114	PNPX	1099	02113	FNPY	1098	02112	[illegible]	1097	[illegible]
PPSIPP	1096	02110	PPSIPR	1095	02107	PPSIPT	1094	02106	PXPHI	1093	02105	FXR	1092	[illegible]
PXTHET	1091	02103	PYPHI	1090	02102	PYR	1089	02101	PYTHET	1088	02100	PZPHI	1087	[illegible]
PZR	1086	02076	PZTHET	1085	02075	Q1	1084	02074	Q2	1083	02073	Q3	1082	[illegible]
Q4	1081	02071	Q5	1080	02070	Q6	1079	02067	R1M	1078	02066	[illegible]	1077	[illegible]
R2M	1076	02064	[illegible]	1075	[illegible]	R4M	1074	02062	R4S	1073	02061	[illegible]	1072	[illegible]

SIGN	1070	02056	SINPSI	1069	02055	T1	1068	02054	TAA	1067	02053
TAC	1065	02051	TAD	1064	02050	TAE	1063	02047	TAF	1062	02046
TE10	1060	02044	TE11	1059	02043	TE12	1058	02042	TE13	1057	02041
TE1	1055	02037	TE2	1054	02036	TE3	1053	02035	TE4	1052	02034
TE6	1050	02032	TE7	1049	02031	TE8	1048	02030	TE9	1047	02027
TEA	1045	02025	TEB	1044	02024	TEC	1043	02023	TED	1042	02022
TEF	1040	02020	TEG	1039	02017	TEH	1038	02016	TEJ	1037	02015
THET1M	1035	02013	THET1S	1034	02012	THET2M	1033	02011	THET2S	1032	02010
Y	1030	02006	X2	1029	02005	X3	1028	02004	[illegible]	1027	02003
PHI	1025	02001	YR	1024	02000	V	1023	01777	VSI	1022	01776
Z	1020	01774									

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
1)	1016	01770	2)	996	01744	3)	999	01747	6)	1008	01760

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

[illegible] SQRT QATAN COS SIN QW (STH) (FIL)

```
00000 ELECTX BCD 1ELECTX
00001 MAGY   BCD 1MAGY
00002 COLFRZ BCD 1COLFRZ
00003 SQRT   BCD 1SQRT
00004 QATAN  BCD 1QATAN
00005 COS    BCD 1COS
00006 SIN    BCD 1SIN
00007 QW     BCD 1QW
00010 (STM)  BCD 1(STM)
00011 (FIL)  BCD 1(FIL)
00012 $      HTR
00013        HTR
00014        HTR
00015        BCD 1RINDEX
00016        SXD $,1
00017        SXD $+1,2
00020        SXD $+2,4
00021 6A     CLA 3)
00022        STO PI
      7A     BSS
0.  2  8,    TSX ELECTX,4
00024        TS  X
00025   .    TSX PXR
00026        TSX PXTHET
00027        TSX PXPHI
00030 9A     CLA X
00031        STO R
00032 10A    CLA PXR
00033        STO R-1
00034 11A    CLA PXTHET
00035        STO R-2
00036 12A    CLA PXPHI
00037        STO R-3
00040 13A    CLA W-66
00041        STO SIGN
      14A    BSS
00042 15A    TSX MAGY,4
00043        TSX Y
00044        TSX YR
00045        TSX YTHETA
00046        TSX YPHI
00047        TSX PYR
00050        TSX PYTHET
00051        TSX PYPHI
00052 16A    CLA Y
00053        STO R-4
00054 17A    CLA YR
00055        STO R-5
00056 18A    CLA YTHETA
00057        STO R-6
00060 19A    CLA YPHI
00061        STO R-7
00062 20A    CLA PYR
00063        STO R-8
00064 21A    CLA PYTHET
00065        STO R-9
00066 22A    CLA PYPHI
00067        STO R-10

       23A   BSS
00070 24A    TSX COLFRZ,4
00071        TSX Z
00072        TSX PZR
00073        TSX PZTHET
00074        TSX PZPHI
00075 25A    CLA Z
00076        STO R-11
00077 26A    CLA PZR
00100        STO R-12
00101 27A    CLA PZTHET
00102        STO R-13
00103 28A    CLA PZPHI
00104        STO R-14
00105 29A    CLA X
00106        STO W-65
00107 30A    CLA Y
00110        STO W-64
00111 31A    CLA Z
00112        STO W-63
00113 32A    LDQ Y
00114        FMP W
 01 5        STO 1)+1
001 6          Q V-8
00117        FMP Y HI
00120        STO 1)+2
00121        LDQ V-7
00122        FMP YTHETA
00123        STO 1)+3
00124        LDQ V-6
00125        FMP YR
00126        FAD 1)+3
00127        FAD 1)+2
00130        FDP  )+1
00131        STQ COSPSI
00132 33A    CLA COSPSI
00133        STO R-15
00134 34A    LDQ COSPSI
00135        FMP COSPSI
00136        CHS
00137        FAD 3)+1
             BSS
00140        TSX SQRT,4
00141        STO SINPSI
00142 35A    CLA SINPSI
00143        STO R-16
00144 36A    LDQ Y
00145        FMP SINPSI
00146        STO YSI
00147 37A    CLA YSI
00150        STQ R-17
00151 38A    LDQ Y
00152        FMP COSPSI
00153        STO YCI
00154 39A    CLA YCI
00155        STO R-18
00156 40A    CLA Z
00157        FSB 3)+2

00160 40A1   TZE 43A
00161        TPL 43A
00162 41A    CLA 3)+3
00163        STO T1
00164 42A    TRA 44A
00165 43A    LDQ Z
00166        FMP Z
00167        STO T1
00170 44A    CLA 3)+1
00171        FSB X
00172        STO 1)+1
00173        LDQ 1)+1
00174        FMP 1)+1
00175        FSB T1
00176        STO TE1
00177 45A    CLA TE1
00200        STO R-19
00201 46A    LDQ YSI
00202        FMP YSI
00203        STO TEE
00204 47A    LDQ YCI
00205        FMP YCI
00206        STO TEA
00207 48A    LDQ TE1
00210        FMP 3)+4
0021         XCA
00212        F P  EA
00213        STO 1)+1
00214        LDQ TEE
00215        FMP TEE
00216        FAD 1)+1
00217        STO TE2
00220 49A    CLA TE2
00221        STO R-20
00222 50A    CLA 3)+1
00223        FSB X
00224        STO 1)+1
00225        LDQ Z
00226        FMP 1)+1
00227        STO TEB
00230 51A    LDQ TEB
00231        FMP 3)+5
00232        XCA
00233        FMP TEA
00234        STO TE3
00235 52A    CLA TE3
00236        STO R-21
00237 53A    LDQ TE3
00240        FMP TE3
00241        STO 1)+1
00242        LDQ TE2
00243        FMP TE2
00244        FAD 1)+1
00245        STO R4S
00246 54A    CLA R4S
             BSS
00247        TSX SQRT,4
00250        STO R2S
```

```
00251 55A   CLA R2S
00252       STO R-22
00253 56A   CLA R2S
            BSS
00254       TSX SQRT,4
00255       STO R1S
00256 57A   CLS TE3
00257       STO 1)+1
            BSS
00260 58A   TSX QATAN,4
00261       TSX TE2
00262       TSX 1)+1
00263       STO THET2S
00264 59A   CLA PI
00265       FSB THET2S
00266 59A1  TZE 60A
00267       TPL 61A
00270 60A   CLS R1S
00271       STO R1S
00272 61A   LDQ 3)+6
00273       FMP THET2S
00274       STO THET1S
00275 62A   CLA R1S
00276       STO R-23
00277 63A   CLA THET2S
00300       STO R-24
00301 64A   CLA THET1S
00302       STO R-25
00303 65A   CLA THET1S
            BSS
00304       TSX COS,4
00305       STO 1)+1
00306       LDQ SIGN
00307       FMP R1S
00310       XCA
00311       FMP 1)+1
00312       STO S1
00313 66A   CLA S1
00314       STO R-26
00315 67A   CLA THET1S
            BSS
00316       TSX SIN,4
00317       STO 1)+1
00320       LDQ SIGN
00321       FMP R1S
00322       XCA
00323       FMP 1)+1
00324       STO S2
00325 68A   CLA S2
00326       STO R-27
00327 69A   LDQ Z
00330       FMP Z
00331       CHS
00332       FSB X
00333       FAD 3)+1
00334       STO 1)+1
00335       LDQ 3)+7
00336       FMP 1)+1

00337       FSB TEE
00340       FAD S1
00341       STO D1
00342 70A   CLA D1
00343       STO R-28
00344 71A   CLA X
00345       FSB 3)+7
00346       STO 1)+1
00347       LDQ 3)+7
00350       FMP Z
00351       XCA
00352       FMP 1)+1
00353       FAD S2
00354       STO D2
00355 72A   CLA D2
00356       STO R-29
00357 73A   LDQ D2
00360       FMP D2
00361       STO 1)+1
00362       LDQ D1
00363       FMP D1
00364       FAD 1)+1
00365       STO TE4
00366 74A   CLA TE4
00367       STO R-30
00370 75A   CLA 3)+1
00371       FSB X
00372       XCA
00373       FMP D2
00374       STO 1)+2
00375       LDQ Z
00376       FMP D1
00377       FAD 1)+2
00400       STO 1)+3
00401       LDQ X
00402       FMP 3)+7
00403       FDP TE4
00404       FMP 1)+3
00405       STO TE5
00406 76A   CLA TE5
00407       STO R-31
00410 77A   CLA 3)+1
00411       FSB X
00412       XCA
00413       FMP D1
00414       STO TEC
00415 78A   LDQ Z
00416       FMP D2
00417       CHS
00420       FAD TEC
00421       STO 1)+1
00422       LDQ X
00423       FMP 3)+7
00424       FDP TE4
00425       FMP 1)+1
00426       CHS
00427       FAD 3)+1
00430       STO TE6

00431 79A   CLA TE6
00432       STO R-32
00433 80A   LDQ TE5
00434       FMP TE5
00435       STO 1)+1
00436       LDQ TE6
00437       FMP TE6
00440       FAD 1)+1
00441       STO R4M
00442 81A   CLA R4M
            BSS
00443       TSX SQRT,4
00444       STO R2M
00445 82A   CLA R2M
00446       STO R-33
00447 83A   CLA R2M
            BSS
00450       TSX SQRT,4
00451       STO R1M
      84A   BSS
00452 85A   TSX QATAN,4
00453       TSX TE6
00454       TSX TE5
00455       STO THET2M
00456 86A   CLA PI
00457       FSB THET2M
00460 86A1  TZE 87A
00461       TPL 88A
00462 87A   CLS R1M
00463       STO R1M
00464 88A   LDQ 3)+6
00465       FMP THET2M
00466       STO THET1M
00467 89A   CLA R1M
00470       STO R-34
00471 90A   CLA THET2M
00472       STO R-35
00473 91A   CLA THET1M
00474       STO R-36
00475 92A   CLA THET1M
            BSS
00476       TSX COS,4
00477       STO 1)+1
00500       LDQ R1M
00501       FMP 1)+1
00502       STO AM1
00503 93A   CLA AM1
00504       STO R-37
00505 94A   CLA THET1M
            BSS
00506       TSX SIN,4
00507       STO 1)+1
00510       LDQ R1M
00511       FMP 1)+1
00512       STO AM2
00513 95A   CLA AM2
00514       STO R-38
00515 96A   CLA AM1
```

```
00516       STO W          00610       XCA            00702       LDQ A4
00517 97A   CLS AM2        00611       FMP S1         00703       FMP A5
00520       STO W-1        00612       FSB 1)+1       00704       STO 1)+2
00521 98A   LDQ AM2        00613       STO TEG        00705       LDQ 3)+7
00522       FMP D2         00614 112A  CLA Q5         00706       FMP X
00523       STO 1)+1       00615       FDP TE9        00707       FSB 3)+1
00524       LDQ AM1        00616       FMP TEG        00710       FSB 1)+2
00525       FMP D1         00617       FAD 3)+1       00711       FAD 1)+1
00526       FSB 1)+1       00620       STO A4         00712       STO TEJ
00527       STO TE7        00621 113A  CLA A4         00713 122A  LDQ A4
00530 99A   CLA TE7        00622       STO R-44       00714       FMP B5
00531       STO R-39       00623 114A  CLA 3)+1       00715       STO 1)+1
00532 100A  LDQ AM2        00624       FSB X          00716       LDQ B4
00533       FMP D1         00625       XCA            00717       FMP A5
00534       STO 1)+1       00626       FMP S2         00720       FAD Z
00535       LDQ AM1        00627       STO 1)+2       00721       FAD 1)+1
00536       FMP D2         00630       LDQ Z          00722       STO TEK
00537       FAD 1)+1       00631       FMP S1         00723 123A  LDQ B0
00540       STO TE8        00632       FAD 1)+2       00724       FMP TEK
00541 101A  CLA TE8        00633       FDP TE9        00725       STO 1)+1
00542       STO R-40       00634       STO TEH        00726       LDQ A0
00543 102A  LDQ TE7        00635 115A  LDQ TEH        00727       FMP TEJ
00544       FMP TE7        00636       FMP Q5         00730       FAD 1)+1
00545       STO TED        00637       FAD Z          00731       STO PNPX
00546 103A  LDQ TE8        00640       STO B4         00732 124A  CLA PNPX
00547       FMP TE8        00641 116A  CLA B4         00733       STO R-48
00550       STO TEF        00642       STO R-45       00734 125A  LDQ SINPSI
00551 104A  CLA TED        00643 117A  LDQ Z          00735       FMP SINPSI
00552       FAD TEF        00644       FMP D2         00736       STO Q5
00553       STO 1)+1       00645       CHS            00737 126A  LDQ COSPSI
00554       CLA TE7        00646       FAD TEC        00740       FMP COSPSI
00555       FDP 1)+1       00647       STO 1)+1       00741       XCA
00556       STO A0         00650       LDQ X          00742       FMP 3)+7
00557 105A  CLA A0         00651       FMP 3)+7       00743       STO Q6
00560       STO R-41       00652       FDP TE4        00744 127A  LDQ Q6
00561 106A  CLA TE8        00653       FMP 1)+1       00745       FMP TE1
00562       FDP TE7        00654       STO A5         00746       STO 1)+1
00563       FMP A0         00655 118A  CLA A5         00747       LDQ TEE
00564       STO B0         00656       STO R-46       00750       FMP Q5
00565 107A  CLA B0         00657 119A  CLA 3)+1       00751       FAD 1)+1
00566       STO R-42       00660       FSB X          00752       STO TE10
00567 108A  LDQ S2         00661       XCA            00753 128A  CLA TE10
00570       FMP S2         00662       FMP D2         00754       STO R-49
00571       STO 1)+1       00663       STO 1)+2       00755 129A  LDQ 3)+7
00572       LDQ S1         00664       LDQ Z          00756       FMP TEB
00573       FMP S1         00665       FMP D1         00757       STO TEB2
00574       FAD 1)+1       00666       FAD 1)+2       00760 130A  LDQ TEB2
00575       STO TE9        00667       STO 1)+3       00761       FMP Q6
00576 109A  CLA TE9        00670       LDQ X          00762       STO TE11
00577       STO R-43       00671       FMP 3)+7       00763 131A  CLA TE11
00600 110A  LDQ 3)+7       00672       FDP TE4        00764       STO R-50
00601       FMP TEA        00673       FMP 1)+3       00765 132A  LDQ TE11
00602       STO Q5         00674       STO B5         00766       FMP S2
00603 111A  LDQ Z          00675 120A  CLA B5         00767       STO 1)+1
00604       FMP S2         00676       STO R-47       00770       LDQ TE10
00605       STO 1)+1       00677 121A  LDQ B4         00771       FMP S1
00606       CLA 3)+1       00700       FMP B5         00772       FSB 1)+1
00607       FSB X          00701       STO 1)+1       00773       FDP TE9
```

```
00774        STO A6
00775 133A   CLA A6
00776        STO R-51
00777 134A   LDQ TE11
01000        FMP S1
01001        STO 1)+1
01002        LDQ TE10
01003        FMP S2
01004        FAD 1)+1
01005        FDP TE9
01006        STQ B6
01007 135A   CLA B6
01010        STO R-52
01011 136A   LDQ B5
01012        FMP A0
01013        STO 1)+1
01014        LDQ B0
01015        FMP A5
01016        FAD 1)+1
01017        STO Q1
01020 137A   LDQ B0
01021        FMP B5
01022        STO 1)+1
01023        LDQ A0
01024        FMP A5
01025        FSB 1)+1
01026        STO Q2
01027 138A   LDQ SINPSI
01030        FMP SINPSI
01031        CHS
01032        FAD A6
01033        STO Q3
01034 139A   LDQ Q2
01035        FMP Q3
01036        STO Q4
01037 140A   LDQ Y
01040        FMP Q1
01041        XCA
01042        FMP B6
01043        STO 1)+1
01044        LDQ Y
01045        FMP Q4
01046        FSB 1)+1
01047        STO PNPY
01050 141A   CLA PNPY
01051        STO R-53
01052 142A   LDQ 3)+7
01053        FMP TEA
01054        STO Q5
01055 143A   CLA Q5
01056        FDP TE9
01057        FMP TEG
01060        STO 1)+1
01061        CLA 3)+7
01062        FSB X
01063        FAD 1)+1
01064        STO A7
01065 144A   CLA A7

01066        STO R-54
01067 145A   LDQ TEH
01070        FMP TEA
01071        FAD Z
01072        STO 1)+1
01073        LDQ 3)+7
01074        FMP 1)+1
01075        STO B7
01076 146A   CLA B7
01077        STO R-55
01100 147A   LDQ B5
01101        FMP B7
01102        STO 1)+1
01103        LDQ A5
01104        FMP A7
01105        CHS
01106        FAD X
01107        FAD 1)+1
01110        STO Q1
01111 148A   LDQ B5
01112        FMP A7
01113        STO 1)+1
01114        LDQ A5
01115        FMP B7
01116        FAD 1)+1
01117        STO Q2
01120 149A   LDQ A0
01121        FMP Q2
01122        STO 1)+1
01123        LDQ B0
01124        FMP Q1
01125        FSB 1)+1
01126        STO PNPZ
01127 150A   CLA PNPZ
01130        STO R-56
01131 151A   LDQ X
01132        FMP X
01133        CHS
01134        FAD 3)+1
01135        STO Q3
01136 152A   CLA 3)+1
01137        FSB X
01140        XCA
01141        FMP S2
01142        XCA
01143        FMP Z
01144        STO 1)+2
01145        LDQ S1
01146        FMP Q3
01147        FSB 1)+2
01150        STO Q4
01151 153A   CLA TEA
01152        FDP TE9
01153        FMP Q4
01154        FAD 3)+1
01155        STO 1)+1
01156        LDQ 3)+7
01157        FMP 1)+1

01160        STO A8
01161 154A   CLA A8
01162        STO R-57
01163 155A   CLA 3)+1
01164        FAD X
01165        XCA
01166        FMP Z
01167        XCA
01170        FMP S1
01171        STO 1)+2
01172        LDQ S2
01173        FMP Q3
01174        FAD 1)+2
01175        STO Q6
01176 156A   CLA Q5
01177        FDP TE9
01200        FMP Q6
01201        STO 1)+1
01202        CLA 3)+7
01203        FAD X
01204        STO 1)+2
01205        LDQ Z
01206        FMP 1)+2
01207        FAD 1)+1
01210        STO B8
01211 157A   CLA B8
01212        STO R-58
01213 158A   LDQ B5
01214        FMP B8
01215        STO 1)+1
01216        LDQ A5
01217        FMP A8
01220        STO 1)+2
01221        LDQ X
01222        FMP 3)+7
01223        XCA
01224        FMP X
01225        FSB 1)+2
01226        FAD 1)+1
01227        STO Q1
       159A  BSS
01230 160A   TSX QW,4
01231        TSX B5
01232        TSX B8
01233        STO 1)+1
             BSS
01234 161A   TSX QW,4
01235        TSX A5
01236        TSX A8
01237        STO 1)+2
             BSS
01240 162A   TSX QW,4
01241        TSX X
01242        TSX X
01243        STO 1)+3
01244        LDQ 3)+7
01245        FMP 1)+3
01246        FSB 1)+2
```

```
01247        FAD 1)+1
01250        STO Q1
01251  163A  LDQ B5
01252        FMP A8
01253        STO 1)+1
01254        LDQ A5
01255        FMP B8
01256        STO 1)+2
01257        LDQ X
01260        FMP Z
01261        FAD 1)+2
01262        FAD 1)+1
01263        STO Q2
01264  164A  LDQ B0
01265        FMP Q2
01266        STO 1)+1
01267        LDQ A0
01270        FMP Q1
01271        FAD 1)+1
01272        FDP W-2
01273        XCA
01274        CHS
01275        STO W-10
01276  165A  CLA V-4
             BSS
01277        TSX COS,4
01300        STO Q1
01301  166A  CLA V-4
             BSS
01302        TSX SIN,4
01303        STO Q2
01304  167A  LDQ Q2
01305        FMP 3)+6
01306        XCA
01307        FMP Q2
01310        STO 1)+1
01311        LDQ Q1
01312        FMP 3)+7
01313        XCA
01314        FMP Q1
01315        FAD 1)+1
01316        STO Q3
01317  168A  CLA 3)+1
01320        FDP Q3
01321        STO TE13
01322  169A  CLA TE13
01323        STO R-59
01324  170A  CLA YSI
01325        SSP
01326        FSB 3)+8
01327  170A1 TZE 213A
01330        TPL 171A
01331        TRA 213A
01332  171A  LDQ 3)+7
01333        FMP TE1
01334        FSB TEE
01335        STO A1
01336  172A  CLA A1

01337        STO R-60
01340  173A  LDQ 3)+4
01341        FMP TEB
01342        STO B1
01343  174A  CLA B1
01344        STO R-61
01345  175A  LDQ B1
01346        FMP S2
01347        STO 1)+1
01350        LDQ A1
01351        FMP S1
01352        FSB 1)+1
01353        FDP TE9
01354        STO 1)+2
01355        CLA 3)+1
01356        FAD 1)+2
01357        STO A2
01360  176A  CLA A2
01361        STO R-62
01362  177A  LDQ A1
01363        FMP S2
01364        STO 1)+1
01365        LDQ S1
01366        FMP B1
01367        FAD 1)+1
01370        FDP TE9
01371        STO B2
01372  178A  CLA B2
01373        STO R-63
01374  179A  LDQ A2
01375        FMP A5
01376        STO TAA
01377  180A  LDQ B2
01400        FMP B5
01401        STO TAB
01402  181A  LDQ A2
01403        FMP B5
01404        STO TAC
01405  182A  LDQ B2
01406        FMP A5
01407        STO TAD
01410  183A  CLA TAA
01411        FSB TAB
01412        STO 1)+1
01413        LDQ A0
01414        FMP 1)+1
01415        STO TAE
01416  184A  CLA TAC
01417        FAD TAD
01420        STO 1)+1
01421        LDQ B0
01422        FMP 1)+1
01423        STO TAF
01424  185A  LDQ YCI
01425        FMP YSI
01426        STO Q1
01427  186A  CLA TAE
01430        FSB TAF

01431        STO 1)+1
01432        LDQ Q1
01433        FMP 1)+1
01434        CHS
01435        STO W-9
01436  187A  LDQ W
01437        FMP W
01440        STO TAG
01441  188A  LDQ TAG
01442        FMP YSI
01443        STO TE12
01444  189A  CLA TE12
01445        STO R-64
01446  190A  LDQ V-6
01447        FMP YCI
01450        STO X1
01451  191A  LDQ W
01452        FMP YR
01453        STO X2
01454  192A  CLA X1
01455        FSB X2
01456        STO X3
01457  193A  CLA W-9
01460        FDP TE12
01461        FMP X3
01462        STO W-3
01463  194A  LDQ V-7
01464        FMP YCI
01465        STO X1
01466  195A  LDQ W
01467        FMP YTHETA
01470        STO X2
01471  196A  CLA X1
01472        FSB X2
01473        STO X3
01474  197A  CLA W-9
01475        FDP TE12
01476        FMP X3
01477        STO W-4
01500  198A  LDQ V-8
01501        FMP YCI
01502        STO X1
01503  199A  LDQ W
01504        FMP YPHI
01505        STO X2
01506  200A  CLA X1
01507        FSB X2
01510        STO X3
01511  201A  CLA W-9
01512        FDP TE12
01513        FMP X3
01514        STO W-5
01515  202A  LDQ V-7
01516        FMP YR
01517        STO 1)+1
01520        LDQ V-6
01521        FMP YTHETA
01522        FSB 1)+1
```

```
01523        STO X1
01524 203A   LDQ W
01525        FMP YSI
01526        STO X2
01527 204A   CLA X1
01530        FDP X2
01531        STO X3
01532 205A   LDQ X3
01533        FMP TE13
01534        STO PPSIPT
01535 206A   CLA PPSIPT
01536        STO R-65
01537 207A   CLA 3)+3
01540        STO PPSIPR
01541 208A   CLA 3)+3
01542        STO PPSIPP
01543 209A   LDQ W-9
01544        FMP PPSIPR
01545        STO 1)+1
01546        LDQ PNPZ
01547        FMP PZR
01550        STO 1)+2
01551        LDQ PNPY
01552        FMP PYR
01553        STO 1)+3
01554        LDQ PNPX
01555        FMP PXR
01556        FAD 1)+3
01557        FAD 1)+2
01560        FAD 1)+1
01561        STO W-6
01562 210A   LDQ W-9
01563        FMP PPSIPT
01564        STO 1)+1
01565        LDQ PNPZ
01566        FMP PZTHET
01567        STO 1)+2
01570        LDQ PNPY
01571        FMP PYTHET
01572        STO 1)+3
01573        LDQ PNPX
01574        FMP PXTHET
01575        FAD 1)+3
01576        FAD 1)+2
01577        FAD 1)+1
01600        STO W-7
01601 211A   LDQ W-9
01602        FMP PPSIPP
01603        STO 1)+1
01604        LDQ PNPZ
01605        FMP PZPHI
01606        STO 1)+2
01607        LDQ PNPY
01610        FMP PYPHI
01611        STO 1)+3
01612        LDQ PNPX
01613        FMP PXPHI
01614        FAD 1)+3

01615        FAD 1)+2
01616        FAD 1)+1
01617        STO W-8
01620 212A   TRA 218A
01621 213A   CLA 3)+3
01622        STO W-3
01623 214A   CLA 3)+3
01624        STO W-4
01625 215A   CLA 3)+3
01626        STO W-5
01627 216A   CLA TE13
01630        STO PPSIPT
01631 217A   TRA 207A
01632 218A   LDQ V-6
01633        FMP V-6
01634        STO X1
01635 219A   LDQ V-7
01636        FMP V-7
01637        STO X2
01640 220A   LDQ V-8
01641        FMP V-8
01642        STO X3
01643 221A   CLA X1
01644        FAD X2
01645        FAD X3
             BSS
01646        TSX SQRT,4
01647        STO TE14
01650 222A   CLA PPSIPR
01651        STO R-66
01652 223A   CLA PPSIPP
01653        STO R-67
01654 224A   CLA TE14
01655        STO R-68
01656 225A   CLA V-6
01657        FDP TE14
01660        FMP W
01661        STO V-6
01662 226A   CLA V-7
01663        FDP TE14
01664        FMP W
01665        STO V-7
01666 227A   CLA V-8
01667        FDP TE14
01670        FMP W
01671        STO V-8
01672 228A   CLA W-67
01673 228A1  TZE 239A
01674        TPL 230A
01675 229A   CLA W
01676        FSB W-68
01677 229A1  TZE 242A
01700        TPL 242A
01701        TRA 233A
01702 230A   CLA V-3
01703        FSB W-67
01704 230A1  TZE 233A
01705        TPL 233A

01706        TRA 242A
01707 231A   CLA W-69
01710        FSB WTST
01711 231A1  TZE 232A
01712        TPL 232A
01713        TRA 242A
01714 232A   CLA WTST
01715        FAD W-62
01716        STO WTST
01717 233A   CAL 2)+1
             BSS
01720        TSX (STH),4
01721        PZE 8)40
01722 234A   LXD 2)+2,1
01723 235A   LDQ R+1,1
01724        STR
01725 235A1  TXI *+1,1,1
01726 235A2  TXL 235A,1,69
      237A   BSS
01727        TSX (FIL),4
01730 238A   TRA 242A
01731 239A   CLA W-69
01732        FSB 3)+1
01733 239A1  TZE 231A
01734        TPL 231A
01735 240A   CLA W-62
01736        STO WTST
01737 241A   TRA 233A
01740 242A   LXD $,1
01741        LXD $+1,2
01742        LXD $+2,4
01743        TRA 1,4
01744 2)     OCT +000002000000
01745        OCT +000006000000
01746        OCT +000001000000
01747 3)     OCT +202622077326
01750        OCT +201400000000
01751        OCT +105447113564
01752        OCT +000000000000
01753        OCT +203400000000
01754        OCT +204400000000
01755        OCT +200400000000
01756        OCT +202400000000
01757        OCT +113715126246
01760 6)     OCT +233000000000
01761        OCT +000000077777
01762        OCT +000000000000
01763        OCT +000001000000
01764        OCT +000000000000
01765        BCD 16)
01766        BCD 1P8E14.
01767 8)40   BCD 1(1H0,1
```

I. SUBROUTINE ELECTX

The whole process of determining the free normalized electron density and its spatial derivatives at any point in space is explored in this subroutine. For the case under consideration this is a simple sphere where the values in Equation 74 are represented by the following components of the W vector

$$W(28) = \mathcal{R}; \quad W(34) = A; \quad W(35) = n$$

```
C     SIMPLE SPHERE ELECTX GIVEN BY NE=A/RANGE**N  DEC.22,1960  IBM-7090
C     WITH INNER SPHERE HAVING NE = CONSTANT FROM RADIUS = W(36) KM
    1 SUBROUTINE ELECTX (X,PXR,PXTHET,PXPHI)
      DIMENSION RECORD(12),V(111),W(250)
      COMMON RECORD,V,W,M
       CALL BIGR
    3 TER1 = ((W(34)/W(3))/W(3))*3.184983E9
    5 IF(W(28)-W(36) )6,11,11
    6 X = TER1/(W(36)**W(35))
    7  PXR = 0.0
    8  PXTHET = 0.0
    9  PXPHI = 0.0
      GO TO 40
   11 ST  = SINF(V(5))
   12 CT = COSF(V(5))
   13  SP = SINF(V(6))
   14  CP = COSF(V(6))
   15  RST = V(4)*ST
   16 RSTCP = RST*CP
   17 RSTSP = RST*SP
   18 RCT = V(4)*CT
   19 XBRP= (W(31)-RSTCP)
   20 YBRP= (W(32)-RSTSP)
   21 ZBRP= (W(33)-RCT)
   22 PARTR = W(35)*(ST*CP*XBRP+ST*SP*YBRP+CT*ZBRP)
   23 PARTT = W(35)*(RCT*(XBRP*CP+YBRP*SP)-RST*ZBRP)
   24 PARTP = W(35)*(YBRP*RSTCP - XBRP*RSTSP)
   26 TER2 = W(28)**W(35)
   27  X = TER1/TER2
   28  TR3 = X/(W(28)**2)
   29  PXR = TR3*PARTR
   30  PXTHET = TR3*PARTT
   31  PXPHI = TR3*PARTP
   40 W(38) =                W(3)*SQRTF(X)
   41 W(39) = (X*W(3)/3.184983E9)*W(3)
   42  RETURN
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                    STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

          DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
       N 32188 76674     RECORD 32561 77461         V 32549 77445         W 32438 77266

           EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

   EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC
     1     1 00000         3     6 00034         5     7 00042         6     8 00046         7     9 00055
     8    10 00057         9    11 00061        11    13 00064        12    14 00067        13    15 00072
    14    16 00075        15    17 00100        16    18 00103        17    19 00106        18    20 00111
    19    21 00114        20    22 00117        21    23 00122        22    24 00125        23    25 00147
    24    26 00170        26    27 00202        27    28 00206        28    29 00211        29    30 00217
    30    31 00222        31    32 00225        40    33 00230        41    34 00236        42    35 00244

                                     STORAGE NOT USED BY PROGRAM

          DEC   OCT              DEC   OCT
          198 00306            32187 76673

                                 LOCATIONS OF NAMES IN TRANSFER VECTOR

          DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
 EXP(3      1 00001      COS       3 00003      BIGR     0 00000      SIN      2 00002      SQRT     4 00004

        STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

          DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
    CT    197 00305      CP      196 00304    PARTP    195 00303    PARTR    194 00302    PARTT    193 00301
   RCT    192 00300   RSTCP      191 00277      RST    190 00276    RSTSP    189 00275       SP    188 00274
    ST    187 00273    TER1      186 00272     TER2    185 00271      TR3    184 00270     XBRP    183 00267
  YBRP    182 00266    ZBRP      181 00265

                  STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

          DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
  1)      176 00260     2)       168 00250     3)      169 00251     6)      171 00253

                        ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
     BIGR       EXP(3       SIN        COS        SQRT
```

```
00000 BIGR    BCD 1BIGR
00001 EXP(3   BCD 1EXP(3
00002 SIN     BCD 1SIN
00003 COS     BCD 1COS
00004 SQRT    BCD 1SQRT
00005 $       HTR
00006         HTR
00007         HTR
00010         BCD 1ELECTX
00011         SXD $,1
00012         SXD $+1,2
00013         SXD $+2,4
00014         CLA 1,4
00015         STA 5A+17
00016         STA 5A+109
00017         STA 5A+113
00020         STA 5A+125
00021         STA 5A+131
00022         CLA 2,4
00023         STA 5A+19
00024         STA 5A+118
00025         CLA 3,4
00026         STA 5A+21
00027         STA 5A+121
00030         CLA 4,4
00031         STA 5A+23
00032         STA 5A+124
        5A    BSS
00033         TSX BIGR,4
00034 6A      CLA W-33
00035         FDP W-2
00036         XCA
00037         FDP W-2
00040         FMP 3)
00041         STO TER1
00042 7A      CLA W-27
00043         FSB W-35
00044 7A1     TZE 13A
00045         TPL 13A
00046 8A      CLA W-35
00047         LDQ W-34
              BSS
00050         TSX EXP(3,4
00051         STO 1)+1
00052         CLA TER1
00053         FDP 1)+1
00054         STO X
00055 9A      CLA 3)+1
00056         STO PXR
00057 10A     CLA 3)+1
00060         STO PXTHET
00061 11A     CLA 3)+1
00062         STO PXPHI
00063 12A     TRA 33A
00064 13A     CLA V-4
              BSS
00065         TSX SIN,4
00066         STO ST
00067 14A     CLA V-4
              BSS
00070         TSX COS,4
00071         STO CT
00072 15A     CLA V-5
              BSS
00073         TSX SIN,4
00074         STO SP
00075 16A     CLA V-5
              BSS
00076         TSX COS,4
00077         STO CP
00100 17A     LDQ V-3
00101         FMP ST
00102         STO RST
00103 18A     LDQ RST
00104         FMP CP
00105         STO RSTCP
00106 19A     LDQ RST
00107         FMP SP
00110         STO RSTSP
00111 20A     LDQ V-3
00112         FMP CT
00113         STO RCT
00114 21A     CLA W-30
00115         FSB RSTCP
00116         STO XBRP
00117 22A     CLA W-31
00120         FSB RSTSP
00121         STO YBRP
00122 23A     CLA W-32
00123         FSB RCT
00124         STO ZBRP
00125 24A     LDQ CT
00126         FMP ZBRP
00127         STO 1)+1
00130         LDQ YBRP
00131         FMP ST
00132         XCA
00133         FMP SP
00134         STO 1)+2
00135         LDQ XBRP
00136         FMP ST
00137         XCA
00140         FMP CP
00141         FAD 1)+2
00142         FAD 1)+1
00143         STO 1)+3
00144         LDQ W-34
00145         FMP 1)+3
00146         STO PARTR
00147 25A     LDQ RST
00150         FMP ZBRP
00151         STO 1)+1
00152         LDQ YBRP
00153         FMP SP
00154         STO 1)+2
00155         LDQ XBRP
00156         FMP CP
00157         FAD 1)+2
00160         STO 1)+3
00161         LDQ RCT
00162         FMP 1)+3
00163         FSB 1)+1
00164         STO 1)+4
00165         LDQ W-34
00166         FMP 1)+4
00167         STO PARTT
00170 26A     LDQ XBRP
00171         FMP RSTSP
00172         STO 1)+1
00173         LDQ YBRP
00174         FMP RSTCP
00175         FSB 1)+1
00176         STO 1)+2
00177         LDQ W-34
00200         FMP 1)+2
00201         STO PARTP
00202 27A     CLA W-27
00203         LDQ W-34
              BSS
00204         TSX EXP(3,4
00205         STO TER2
00206 28A     CLA TER1
00207         FDP TER2
00210         STQ X
00211 29A     LDQ W-27
00212         FMP W-27
00213         STO 1)+1
00214         CLA X
00215         FDP 1)+1
00216         STQ TR3
00217 30A     LDQ TR3
00220         FMP PARTR
00221         STO PXR
00222 31A     LDQ TR3
00223         FMP PARTT
00224         STO PXTHET
00225 32A     LDQ TR3
00226         FMP PARTP
00227         STO PXPHI
00230 33A     CLA X
              BSS
00231         TSX SQRT,4
00232         STO 1)+1
00233         LDQ W-2
00234         FMP 1)+1
00235         STO W-37
00236 34A     CLA X
00237         FDP 3)
00240         FMP W-2
00241         XCA
00242         FMP W-2
00243         STO W-38
00244 35A     LXD $,1
00245         LXD $+1,2
```

J. SUBROUTINE BIGR

This subroutine is used to calculate the distance from the center of the ionizing source to the spatial point r, θ, φ at which the electron density and spatial gradients are desired. For the simple sphere this distance is W(28) = $\mathcal{R}$. In addition to this it calculates the angle that $\mathcal{R}$ makes with respect to the vertical distance passing through the center of the ionizing source.

```
C     SUBROUTINE   BIGR   DECEMBER 22,1960    IBM-7090
 1    SUBROUTINE BIGR
      COMMON RECORD,V,W,N
      DIMENSION RECORD(12),V(111),W(250)
 4    RST=V(4)*SINF(V(5))
 5    RSTCP=RST*COSF(V(6))
 6    RSTSP=RST*SINF(V(6))
 7    RCT=V(4)*COSF(V(5))
 8    W(28)=SQRTF((W(31)-RSTCP)**2+(W(32)-RSTSP)**2+(W(33)-RCT)**2)
 9    W(55)=SQRTF((W(52)-RSTCP)**2+(W(53)-RSTSP)**2+(W(54)-RCT)**2)
10    TERM1=(W(30)**2+(W(28))**2-(V(4))**2)/(2.0*W(28)*W(30))
11    IF(ABSF(TERM1)-1.0E-7)12,12,14
12    W(29)=0.0
13    GO TO 15
14    W(29)= -(TERM1)
15    TE2 =((V(4))**2-(W(19))**2-(W(55))**2)/(2.0*W(19)*W(55))
      TE3 = SQRTF(1.0 -TE2**2)
16    W(37) = ARCOS(TE3)
17    RETURN
      END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                    STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

     DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
 N 32138 76674       RECORD 32561 77461         V 32549 77445         W 32438 77266

          EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

 EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC
   1     1 00000         4     5 00013         5     6 00021         6     7 00027         7     8 00035
   8     9 00043         9    10 00070        10    11 00115        11    12 00136        12    13 00143
  13    14 00145        14    15 00146        15    16 00150        16    18 00177        16    19 00177
  17    20 00202

                              STORAGE NOT USED BY PROGRAM

     DEC   OCT              DEC   OCT
     155 00233            32187 76673

                              LOCATIONS OF NAMES IN TRANSFER VECTOR

     DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
 COS   1 00001       ARCOS    3 00003       SIN    0 00000       SQRT    2 00002

        STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

     DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
 RCT 154 00232       RSTCP  153 00231       RST  152 00230       RSTSP  151 00227       TE2  150 00226
 TE3 149 00225       TERM1  148 00224

                    STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

     DEC   OCT              DEC   OCT             DEC   OCT             DEC   OCT             DEC   OCT
1)   144 00220      2)      134 00206     3)      135 00207     6)      139 00213

                      ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
 SIN       COS       SQRT      ARCOS
```

```
00000 SIN    BCD 1SIN
00001 COS    BCD 1COS
00002 SQRT   BCD 1SQRT
00003 ARCOS  BCD 1ARCOS
00004 $      HTR
00005        HTR
00006        HTR
00007        BCD 1BIGR
00010        SXD $,1
00011        SXD $+1,2
00012        SXD $+2,4
00013 5A     CLA V-4
             BSS
00014        TSX SIN,4
00015        STO 1)+1
00016        LDQ V-3
00017        FMP 1)+1
00020        STO RST
00021 6A     CLA V-5
             BSS
00022        TSX COS,4
00023        STO 1)+1
00024        LDQ RST
00025        FMP 1)+1
00026        STO RSTCP
00027 7A     CLA V-5
             BSS
00030        TSX SIN,4
00031        STO 1)+1
00032        LDQ RST
00033        FMP 1)+1
00034        STO RSTSP
00035 8A     CLA V-4
             BSS
00036        TSX COS,4
00037        STO 1)+1
00040        LDQ V-3
00041        FMP 1)+1
00042        STO RCT
00043 9A     CLA W-32
00044        FSB RCT
00045        STO 1)
00046        LDQ 1)
00047        FMP 1)
00050        STO 1)+1
00051        CLA W-31
00052        FSB RSTSP
00053        STO 1)
00054        LDQ 1)
00055        FMP 1)
00056        STO 1)+2
00057        CLA W-30
00060        FSB RSTCP
00061        STO 1)
00062        LDQ 1)
00063        FMP 1)
00064        FAD 1)+2
00065        FAD 1)+1
```

```
             BSS
00066        TSX SQRT,4
00067        STO W-27
00070 10A    CLA W-53
00071        FSB RCT
00072        STO 1)
00073        LDQ 1)
00074        FMP 1)
00075        STO 1)+1
00076        CLA W-52
00077        FSB RSTSP
00100        STO 1)
00101        LDQ 1)
00102        FMP 1)
00103        STO 1)+2
00104        CLA W-51
00105        FSB RSTCP
00106        STO 1)
00107        LDQ 1)
00110        FMP 1)
00111        FAD 1)+2
00112        FAD 1)+1
             BSS
00113        TSX SQRT,4
00114        ST  W-5
0'11  11A    LDQ W-29
00116        FMP 3)
00117        XCA
00120        FMP W-27
00121        STO 1)+1
00122        LDQ V-3
00123        FMP V-3
00124        STO 1)+2
00125        LDQ W-27
00126        FMP W-27
00127        STO 1)+3
00130        LDQ W-29
00131        FMP W-29
00132        FAD 1)+3
00133        FSB 1)+2
00134        FDP 1)+1
00135        STQ TERM1
00136 12A    CLA TERM1
00137        SSP
00140        FSB 3)+1
00141 12A1   TZE 13A
00142        TPL 15A
00143 13A    CLA 3)+2
00144        STO W-28
00145 14A    TRA 16A
00146 15A    CLS TERM1
00147        STO W-28
00150 16A    LDQ W-54
00151        FMP 3)
00152        XCA
00153        FMP W-18
00154        STO 1)+1
00155        LDQ W-54
```

```
00156        FMP W-54
00157        STO 1)+2
00160        LDQ W-18
00161        FMP W-18
00162        STO 1)+3
00163        LDQ V-3
00164        FMP V-3
00165        FSB 1)+3
00166        FSB 1)+2
00167        FDP 1)+1
00170        STQ TE2
00171 17A    LDQ TE2
00172        FMP TE2
00173        CHS
00174        FAD 3)+3
             BSS
00175        TSX SQRT,4
00176        STO TE3
      18A    BSS
00177 19A    TSX ARCOS,4
00200        TSX TE3
00201        STO W-56
00202 20A    LXD $,
00'03        LX  $+1,2
00204        LXD $+2,4
00205        TRA 1,4
00206 2)     OCT +000002000000
00207 3)     OCT +202400000000
00210        OCT +151655376246
00211        OCT +000000000000
00212        OCT +201400000000
00213 6)     OCT +233000000000
00214        OCT +000000077777
00215        OCT +000000000000
00216        OCT +000001000000
00217        OCT +000000000000
```

K. SUBROUTINE MAGY

The normalized earth's magnetic field and its spatial derivatives are calculated in this subroutine. In this program it is represented by an assumed magnetic dipole field with the dipole located at the center of the earth.

```
C      SUBROUTINE MAGY   DECEMBER 22.1960   IBM-7090
       SUBROUTINE MAGY( V,VR,VTHETA,VPHI,DVDR,DVDTHE,DVDPHI)
       DIMENSION RECORD(12),V(111),W(250)
       COMMON  RECORD,V,W,N
       TERM7 = SINF( V(5))
       TERM8 = COSF( V(5))
       TERM9 = SQRTF (1.0 + 3.0* TERM8**2   )
       TERM10=(W(19)/V(4))**3
       T1 = W(25)*TERM10/W(3)
       V = T1* TERM9
       VR =  - 2.0 * T1 * TERM8
       VTHETA = - T1 * TERM7
       VPHI = 0.0
       DVDR=  (-3.0*V )/V(4)
       DVDTHE =  (-3.0*V*TERM7*TERM8)/(TERM9**2)
       DVDPHI = 0.0
       RETURN
       END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                    STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

       DEC   OCT                 DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
     N 32188 76674        RECORD 32561 77461          V 32549 77445          W 32438 77266

                                  STORAGE NOT USED BY PROGRAM

       DEC   OCT                 DEC   OCT
       107 00153               32187 76673

                                LOCATIONS OF NAMES IN TRANSFER VECTOR

       DEC   OCT                 DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
  COS    1 00001           SIN     0 00000      SQRT     2 00002

       STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

       DEC   OCT                 DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
    T1  106 00152      TERM10   105 00151    TERM7   104 00150    TERM8   103 00147    TEPM9   102 00146

                   STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

       DEC   OCT                 DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
 1)    100 00144         2)       89 00131     3)       91 00133     6)       95 00137

                         ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
  SIN        COS         SQRT
```

```
00000 SIN    BCD 1SIN              00067         FMP 3)+2
00001 COS    BCD 1COS              00070         XCA
00002 SQRT   BCD 1SQRT             00071         FMP T1
00003 S      HTR                   00072         CHS
00004        HTR                   00073         STO YR
00005        HTR                   00074 12A     LDQ T1
00006        BCD 1MAGY             00075         FMP TERM7
00007        SXD S,1               00076         CHS
00010        SXD S+1,2             00077         STO YTHETA
00011        SXD S+2,4             00100 13A     CLA 3)+3
00012        CLA 1,4               00101         STO YPHI
00013        STA 5A+27             00102 14A     LDQ 3)+1
00014        STA 5A+41             00103         FMP Y
00015        STA 5A+51             00104         CHS
00016        CLA 2,4               00105         FDP V-3
00017        STA 5A+33             00106         STQ DYDR
00020        CLA 3,4               00107 15A     LDQ TERM9
00021        STA 5A+37             00110         FMP TERM9
00022        CLA 4,4               00111         STO 1)+1
00023        STA 5A+39             00112         LDQ TERM8
00024        CLA 5,4               00113         FMP 3)+1
00025        STA 5A+44             00114         XCA
00026        CLA 6,4               00115         FMP Y
00027        STA 5A+56             00116         XCA
00030        CLA 7,4               00117         FMP TERM7
00031        STA 5A+58             00120         CHS
00032 5A     CLA V-4               00121         FDP 1)+1
             BSS                   00122         STQ DYDTHE
00033        TSX SIN,4             00123 16A     CLA 3)+3
00034        STO TERM7             00124         STO DYDPHI
00035 6A     CLA V-4               00125 17A     LXD S,1
             BSS                   00126         LXD S+1,2
00036        TSX COS,4             00127         LXD S+2,4
00037        STO TERM8             00130         TRA 8,4
00040 7A     LDQ TERM8             00131 2)      OCT +000002000000
00041        FMP TERM8             00132         OCT +000003000000
00042        STO 1)+1              00133 3)      OCT +201400000000
00043        LDQ 3)+1              00134         OCT +202600000000
00044        FMP 1)+1              00135         OCT +202400000000
00045        FAD 3)                00136         OCT +000000000000
             BSS                   00137 6)      OCT +233000000000
00046        TSX SQRT,4            00140         OCT +000000077777
00047        STO TERM9             00141         OCT +000000000000
00050 8A     CLA W-18              00142         OCT +000001000000
00051        FDP V-3               00143         OCT +000000000000
00052        STO 1)
00053        FMP 1)
00054        LRS 35
00055        FMP 1)
00056        STO TERM10
00057 9A     CLA W-24
00060        FDP W-2
00061        FMP TERM10
00062        STO T1
00063 10A    LDQ T1
00064        FMP TERM9
00065        STO Y
00066 11A    LDQ TERM8
```

L. SUBROUTINE COLFRZ

The normalized collision frequency and its derivatives at a spatial point are determined in this subroutine. In this presentation it is assumed that collision frequency varies exponentially with height. One should refer to the definition of the W vector where the required coefficients for the height stratifications are defined.

```
C       SUBROUTINE COLFRZ   DECEMBER 22,1960    IBM-7090
C       W(19) + 100 = 6.459112E3 , W(19) + 200 = 6.5569112E3
C       W(19) + 300 = 6.6569112E3 ,  W(19) + 400 = 6.7569112E3
  1     SUBROUTINE  COLFRZ (Z, PZR, PZTHET,PZPHI)
        DIMENSION RECORD(12),V(111),W(250)
        COMMON RECORD,V,W,N
        IF(V(4) - 6.4569112E3)2,2,9
  2     A = W(27)
        B = W(26)
  3     TERM1 = -A*(V(4)-W(19))
        Z = B*EXPF(TERM1)/W(3)
  4     PZR = -A*Z
  5      PZTHET = 0.0
  6     PZPHI  = 0.0
        GO TO 7
  9     IF(V(4) - 6.5569112E3)12,12,10
 12     A = W(71)
        B = W(72)
        GO TO 3
 10     IF(V(4) - 6.6569112E3)13,13,11
 13     A = W(73)
        B = W(74)
        GO TO 3
 11     IF(V(4) - 6.7569112E3)14,14,15
 14     A = W(75)
        B = W(76)
        GO TO 3
 15     Z = 0.0
        PZR = 0.0
        GO TO 5
  7      RETURN
        END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                    STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

       DEC   OCT              DEC   OCT            DEC   OCT           DEC   OCT          DEC   OCT
     N 32188 76674     RECORD 32561 77461        V 32549 77445       W 32438 77266

          EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

   EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC       EFN   IFN   LOC
     1     1 00000         2     6 00027         3     8 00033         4    10 00047         5    11 00053
     6    12 00055         9    14 00060        12    15 00064        10    18 00071        13    19 00075
    11    22 00102        14    23 00106        15    26 00113         7    29 00120

                                 STORAGE NOT USED BY PROGRAM

       DEC   OCT              DEC   OCT
       100 00144            32187 76673

                            LOCATIONS OF NAMES IN TRANSFER VECTOR

       DEC   OCT              DEC   OCT            DEC   OCT           DEC   OCT          DEC   OCT
   EXP   0 00000

    STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

       DEC   OCT              DEC   OCT            DEC   OCT           DEC   OCT          DEC   OCT
   B    99 00143          A    98 00142     TERM1  97 00141

                      STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

       DEC   OCT              DEC   OCT            DEC   OCT           DEC   OCT          DEC   OCT
 1)     95 00137     2)       84 00124     3)      85 00125     6)     90 00132

                       ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
   EXP
```

```
00000 EXP   BCD 1EXP
00001 $     HTR
00002       HTR
00003       HTR
00004       BCD 1COLFRZ
00005       SXD $,1
00006       SXD $+1,2
00007       SXD $+2,4
00010       CLA 1,4
00011       STA 5A+19
00012       STA 5A+21
00013       STA 5A+57
00014       CLA 2,4
00015       STA 5A+23
00016       STA 5A+59
00017       CLA 3,4
00020       STA 5A+25
00021       CLA 4,4
00022       STA 5A+27
00023 5A    CLA V-3
00024       FSB 3D
00025 5A1   TZE 6A
00026       TPL 14A
00027 6A    CLA W-26
00030       STO A
00031 7A    CLA W-25
00032       STO B
00033 8A    CLA V-3
00034       FSB W-19
00035       STO 1D+1
00036       LDQ A
00037       FMP 1D+1
00040       CHS
00041       STO TERM1
00042 9A    CLA TERM1
            BSS
00043       TSX EXP,4
00044       FDP W-2
00045       FMP B
00046       STO Z
00047 10A   LDQ A
00050       FMP Z
00051       CHS
00052       STO PZR
00053 11A   CLA 3D+1
00054       STO PZTHET
00055 12A   CLA 3D+1
00056       STO PZPHI
00057 13A   TRA 29A
00060 14A   CLA V-3
00061       FSB 3D+2
00062 14A1  TZE 15A
00063       TPL 18A
00064 15A   CLA W-70
00065       STO A
00066 16A   CLA W-71
00067       STO B
00070 17A   TRA 8A
00071 18A   CLA V-3
00072       FSB 3D+3
00073 18A1  TZE 19A
00074       TPL 22A
00075 19A   CLA W-72
00076       STO A
00077 20A   CLA W-73
00100       STO B
00101 21A   TRA 8A
00102 22A   CLA V-3
00103       FSB 3D+4
00104 22A1  TZE 23A
00105       TPL 26A
00106 23A   CLA W-74
00107       STO A
00110 24A   CLA W-75
00111       STO B
00112 25A   TRA 8A
00113 26A   CLA 3D+1
00114       STO Z
00115 27A   CLA 3D+1
00116       STO PZP
00117 28A   TRA 11A
00120 29A   LXD $,1
00121       LXD $+1,2
00122       LXD $+2,4
00123       TRA 5,4
00124 2D    OCT +000000000000
00125 3D    OCT +215323435121
00126       OCT +000000000000
00127       OCT +215631635121
00130       OCT +215640035121
00131       OCT +215646235121
00132 6D    OCT +233000000000
00133       OCT +000000077777
00134       OCT +000000000000
00135       OCT +000001000000
00136       OCT +000000000000
```

M. SUBROUTINE RCOORD

This subroutine permits the transformation from the earth centered geomagnetic spherical coordinate system, to the radar coordinate system.

```
C      SUBROUTINE  RCOORD     JANUARY 18,1961  IBM-7090
       SUBROUTINE RCOORD
       COMMON RECORD,V,W,M,XN,YN,ZN,G
       DIMENSION RECORD(12),V(111),W(250)
       DIMENSION XN(7),YN(7),ZN(8),G(3,3)
       RST = V(4)*SINF(V(5))
       XP = RST*COSF(V(6)) - XN(7)
       YP = RST*SINF(V(6)) - YN(7)
       ZP = V(4)*COSF(V(5)) - ZN(7)
       EPSN = XP*XN(1) + ZP*XN(2) + YP*XN(3) -(ZP*XN(4) + YP*XN(5) +
      1XP*XN(6))
       ETAN = YP*YN(1) + XP*YN(2) + ZP*YN(3) - ( YP*YN(4) + XP*YN(5)  +
      1ZP*YN(6))
       ZETAN = ZP*ZN(1) + YP*ZN(2) + XP*ZN(3) - ( XP*ZN(4) + ZP*ZN(5) +
      1YP*ZN(6))
       EPS = EPSN/ZN(8)
       ETA  =  ETAN/ZN(8)
       ZETA = ZETAN/ZN(8)
       RCE2 = ETA**2 + ZETA**2
       RCE = SQRTF(RCE2)
       W(60) = SQRTF(RCE2 + EPS**2)
       IF(W(60))10,40,10
10     ANGE = ARCOS(RCE/W(60))
       IF(RCE)20,40,20
20     ANGA = ARCOS(ZETA/RCE)
       W(61) = ANGA*57.29578
       W(62) = ANGE*57.29578
       W(80) = (W(18) -ANGA)*57.29578
       W(81) = (W(17) - ANGE)*57.29578
       W(82) =  V(10)- W(60)
       W(83) = 2.99791E5*V(11) - W(60)
40     RETURN
       END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                    STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

         DEC   OCT            DEC   OCT              DEC   OCT            DEC   OCT            DEC   OCT
       G 32165 76645        N 32188 76674     RECORD 32561 77461        V 32549 77445        W 32438 77266
      XN 32187 76673       YN 32180 76664         ZN 32173 76655

            EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

    EFN   IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC
     10    20 00205         10    21 00210         20    23 00215         20    24 00220         40    31 00252

                                     STORAGE NOT USED BY PROGRAM

         DEC   OCT            DEC   OCT
         202 00312          32156 76634

                                 LOCATIONS OF NAMES IN TRANSFER VECTOR

         DEC   OCT            DEC   OCT              DEC   OCT            DEC   OCT            DEC   OCT
    COS    1 00001     ARCOS    3 00003      SIN       0 00000     SQRT     2 00002

      STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

         DEC   OCT            DEC   OCT              DEC   OCT            DEC   OCT            DEC   OCT
    ETA  201 00311     ETAN   200 00310      EPS     199 00307     EPSN   198 00306     ANGE   197 00305
   ANGA  196 00304     RCE2   195 00303      RCE     194 00302     RST    193 00301       XP   192 00300
     YP  191 00277     ZETAN  190 00276      ZETA    189 00275       ZP   188 00274

                      STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

         DEC   OCT            DEC   OCT              DEC   OCT            DEC   OCT            DEC   OCT
  1)     182 00266    2)      174 00256    3)        175 00257    6)      177 00261

                        ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
    SIN        COS        SQRT       ARCOS
```

```
00000 SIN    BCD 1SIN
00001 COS    BCD 1COS
00002 SQRT   BCD 1SQRT
00003 ARCOS  BCD 1ARCOS
00004 $      HTR
00005        HTR
00006        HTR
00007        BCD 1RCOORD
00010        SXD $,1
00011        SXD $+1,2
00012        SXD $+2,4
00013 6A     CLA V-4
             BSS
00014        TSX SIN,4
00015        STO 1)+1
00016        LDQ V-3
00017        FMP 1)+1
00020        STO RST
00021 7A     CLA V-5
             BSS
00022        TSX COS,4
00023        STO 1)+1
00024        LDQ RST
00025        FMP 1)+1
00026        FSB XN-6
00027        STO XP
00030 8A     CLA V-5
             BSS
00031        TSX SIN,4
00032        STO 1)+1
00033        LDQ RST
00034        FMP 1)+1
00035        FSB YN-6
00036        STO YP
00037 9A     CLA V-4
             BSS
00040        TSX COS,4
00041        STO 1)+1
00042        LDQ V-3
00043        FMP 1)+1
00044        FSB ZN-6
00045        STO ZP
00046 10A    LDQ XP
00047        FMP XN-5
00050        STO 1)+1
00051        LDQ YP
00052        FMP XN-4
00053        STO 1)+2
00054        LDQ ZP
00055        FMP XN-3
00056        FAD 1)+2
00057        FAD 1)+1
00060        STQ 1)+3
00061        LDQ YP
00062        FMP XN-2
00063        STO 1)+4
00064        LDQ ZP
00065        FMP XN-1
00066        STO 1)+5
00067        LDQ XP
00070        FMP XN
00071        FAD 1)+5
00072        FAD 1)+4
00073        FSB 1)+3
00074        STO EPSN
00075 11A    LDQ ZP
00076        FMP YN-5
00077        STO 1)+1
00100        LDQ XP
00101        FMP YN-4
00102        STO 1)+2
00103        LDQ YP
00104        FMP YN-3
00105        FAD 1)+2
00106        FAD 1)+1
00107        STO 1)+3
00110        LDQ ZP
00111        FMP YN-2
00112        STO 1)+4
00113        LDQ XP
00114        FMP YN-1
00115        STO 1)+5
00116        LDQ YP
00117        FMP YN
00120        FAD 1)+5
00121        FAD 1)+4
00122        FSB 1)+3
00123        STO ETAN
00124 12A    LDQ YP
00125        FMP ZN-5
00126        STO 1)+1
00127        LDQ ZP
00130        FMP ZN-4
00131        STO 1)+2
00132        LDQ XP
00133        FMP ZN-3
00134        FAD 1)+2
00135        FAD 1)+1
00136        STO 1)+3
00137        LDQ XP
00140        FMP ZN-2
00141        STO 1)+4
00142        LDQ YP
00143        FMP ZN-1
00144        STO 1)+5
00145        LDQ ZP
00146        FMP ZN
00147        FAD 1)+5
00150        FAD 1)+4
00151        FSB 1)+3
00152        STO ZETAN
00153 13A    CLA EPSN
00154        FDP ZN-7
00155        STQ EPS
00156 14A    CLA ETAN
00157        FDP ZN-7
00160        STQ ETA
00161 15A    CLA ZETAN
00162        FDP ZN-7
00163        STQ ZETA
00164 16A    LDQ ZETA
00165        FMP ZETA
00166        STO 1)+1
00167        LDQ ETA
00170        FMP ETA
00171        FAD 1)+1
00172        STO RCE2
00173 17A    CLA RCE2
             BSS
00174        TSX SQRT,4
00175        STO RCE
00176 18A    LDQ EPS
00177        FMP EPS
00200        FAD RCE2
             BSS
00201        TSX SQRT,4
00202        STO W-59
00203 19A    CLA W-59
00204 19A1   TZE 31A
00205 20A    CLA RCE
00206        FDP W-59
00207        STQ 1)+1
             BSS
00210 21A    TSX ARCOS,4
00211        TSX 1)+1
00212        STO ANGE
00213 22A    CLA RCE
00214 22A1   TZE 31A
00215 23A    CLA ZETA
00216        FDP RCE
00217        STQ 1)+1
             BSS
00220 24A    TSX ARCOS,4
00221        TSX 1)+1
00222        STO ANGA
00223 25A    LDQ ANGA
00224        FMP 3)
00225        STO W-60
00226 26A    LDQ ANGE
00227        FMP 3)
00230        STO W-61
00231 27A    CLA W-17
00232        FSB ANGA
00233        XCA
00234        FMP 3)
00235        STO W-79
00236 28A    CLA W-16
00237        FSB ANGE
00240        XCA
00241        FMP 3)
00242        STO W-80
00243 29A    CLA V-9
00244        FSB W-59
00245        STO W-81
```

```
00246 30A    LDQ 3)+1
00247        FMP V-10
00250        FSB W-59
00251        STO W-82
00252 31A    LXD S,1
00253        LXD S+1,2
00254        LXD S+2,4
00255        TRA 1,4
00256 2)     OCT +000002000000
00257 3)     OCT +206712273407
00260        OCT +223444607400
00261 6)     OCT +233000000000
00262        OCT +000000077777
00263        OCT +000000000000
00264        OCT +000001000000
00265        OCT +000000000000
```

N. SUBROUTINE OUTONE

In this presentation this subroutine is used to list the input data and the initial conditions that define the ray tracing problem on Tape Unit 6. Table 5-1 illustrates this output by the statement of the input RECORD and the next ninety-one words of data which define in order the first 70 components of the W vector, followed by the first 21 components of the V vector. This subroutine can be greatly improved.

```
C       SUBROUTINE  OUTONE  DECEMBER 22,1960    IBM-7090
    1   SUBROUTINE  OUTONE (SIGN,ID)
        COMMON RECORD,V,W,N
        DIMENSION RECORD(12),V(111),W(250)
    2   IF(SIGN) 3,3,6
    3   WRITE OUTPUT TAPE 6,4,ID,(RECORD(I),I=1,12)
    4   FORMAT(25H1   EXTRA-ORDINARY RAY    ,I6,12A6)
    5   GO TO 8
    6   WRITE OUTPUT TAPE 6,7,ID,(RECORD(I),I=1,12)
    7   FORMAT(22H1      ORDINARY RAY      ,I6,12A6)
    8   WRITE OUTPUT TAPE 6,9,(W(I),I=1,70),(V(I),I=1,21)
    9   FORMAT (1H0, 1P8E14.7)
        RETURN
        END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                    STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

       DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
   N 32188 76674       RECORD 32561 77461          V 32549 77445          W 32438 77266

         EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

 EFN   IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC        EFN   IFN   LOC
   1     1 00000          4     4 00000          7     5 00000          9     6 00000          2     8 00016
   3     9 00021          3    14 00033          5    15 00034          6    16 00035          6    21 00047
   8    22 00050          8    29 00065

                                   STORAGE NOT USED BY PROGRAM

       DEC   OCT              DEC   OCT
        81 00121            32187 76673

                               LOCATIONS OF NAMES IN TRANSFER VECTOR

       DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
 (FIL)   1 00001      (STA)     0 00000

                      STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

       DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
 8)9    67 00103      8)7      73 00111      8)4      80 00120      2)       58 00072      6)       60 00074

                             ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.

 (STA)       (FIL)
```

```
00000 (STH)  BCD 1(STH)
00001 (FIL)  BCD 1(FIL)
00002 $      HTR
00003        HTR
00004        HTR
00005        BCD 1OUTONE
00006        SXD $,1
00007        SXD $+1,2
00010        SXD $+2,4
00011        CLA 1,4
00012        STA 8A
00013        CLA 2,4
00014        STA 8A+6
00015        STA 8A+18
00016 8A     CLA SIGN
00017 8A1    TZE 9A
00020        TPL 16A
00021 9A     CAL 2)
             BSS
00022        TSX (STH),4
00023        PZE 8)4
00024 10A    LDQ ID
00025        STR
00026 11A    LXD 2)+1,1
00027 12A    LDQ RECORD+1,1
00030        STR
00031 12A1   TXI *+1,1,1
00032 12A2   TXL 12A,1,12
      14A    BSS
00033        TSX (FIL),4
00034 15A    TRA 22A
00035 16A    CAL 2)
             BSS
00036        TSX (STH),4
00037        PZE 8)7
00040 17A    LDQ ID
00041        STR
00042 18A    LXD 2)+1,1
00043 19A    LDQ RECORD+1,1
00044        STR
00045 19A1   TXI *+1,1,1
00046 19A2   TXL 19A,1,12
      21A    BSS
00047        TSX (FIL),4
00050 22A    CAL 2)
             BSS
00051        TSX (STH),4
00052        PZE 8)9
00053 23A    LXD 2)+1,1
00054 24A    LDQ W+1,1
00055        STR
00056 24A1   TXI *+1,1,1
00057 24A2   TXL 24A,1,70
00060 26A    LXD 2)+1,1
00061 27A    LDQ V+1,1
00062        STR
00063 27A1   TXI *+1,1,1
00064 27A2   TXL 27A,1,21
      29A    BSS
00065        TSX (FIL),4
00066 30A    LXD $,1
00067        LXD $+1,2
00070        LXD $+2,4
00071        TRA 3,4
00072 2)     OCT +000006000000
00073        OCT +000001000000
00074 6)     OCT +233000000000
00075        OCT +000000077777
00076        OCT +000000000000
00077        OCT +000001000000
00100        OCT +000000000000
00101        BCD 14.7)
00102        BCD 1 1P8E1
00103 8)9    BCD 1 (1HO,
00104        BCD 1,12A6)
00105        BCD 1    ,I6
00106        BCD 1 RAY
00107        BCD 1DINARY
00110        BCD 1     OR
00111 8)7    BCD 1(22H1
00112        BCD 16)
00113        BCD 1I6,12A
00114        BCD 1AY    ,
00115        BCD 1NARY R
00116        BCD 1A-ORDI
00117        BCD 1   EXTR
00120 8)4    BCD 1(25H1
```

O. SUBROUTINE OUTPUT

This subroutine will presently yield the output data on Tape Unit 6 that is summarized in Tables 5 and 6. Following the information that is the result of subroutine OUTONE, Tables 5-1 and 5-2 illustrate the information obtainable after each integration. Beginning with the first word this information has the following meaning:

Integration number W(70)
Vector components V(2), V(3)
Height above earth surface (km)
Angle θ in degrees
Angle φ in degrees
Vector components V(7) through V(21)
Vector components W(1) through W(11)
Vector components W(28), W(29), W(38), W(39)
Vector components W(55) through W(70)
Vector components W(80) through W(90)

Under certain conditions of vector components W(68) and W(69) the R vector described in subroutine RINDEX will be listed between each of these sets of data for each time that the RINDEX subroutine is entered.

In addition to this data, the data summarized in Table 6 is listed on Tape Unit 10.

```
C      SUBROUTINE  OUTPUT   JANUARY 21, 1961     IBM-7090
 1     SUBROUTINE OUTPUT
       COMMON RECORD,V,W,N,XN,YN,ZN,G
       DIMENSION  RECORD(12),V(111),W(250)
       DIMENSION XN(7),YN(7),ZN(8),G(3,3)
       CALL RCOORD
       IF(W(70)) 3,3,4
 3     II = 0
 4     W(70) = W(70) + 1.0
       IN = W(70)
       W(59) = ( 2.0 * W(1) * W(2) )/ (W(1) **2 - W(2) **2 )
 5     TE1 = V(5)*57.29578
 6     TE2 = V(6)*57.29578
       TE3 = V(4)-W(19)
       W(56)=(2.99791E5*V(11))-W(55)
       W(58)=(W(17)-W(57))*57.29578
        W(57)=(W(57))*57.29579
 7     WRITE OUTPUT TAPE 6,8,W(70),V(2),V(3),TE3,TE1,TE2,V(7),V(8)
 8     FORMAT (1H ,  1P8E14.7)
       WRITE OUTPUT TAPE 6,50
 50    FORMAT(1H )
 9    0WRITE OUTPUT TAPE 6, 9,(V(I),I=9,21),(W(I),I=1,11),W(28),W(29),
      1W(38),W(39),(W(I),I=55,70),(W(I),I=80,90)
       IF(II)10,10,30
 10    WRITE OUTPUT TAPE 10,11,(RECORD(I),I=1,12)
       WRITE OUTPUT TAPE  6,11,(RECORD(I),I=1,12)
 11    FORMAT(1H1,23X,12A6)
       WRITE OUTPUT TAPE 10,12
 12     FORMAT(5H0  IN,5X,6HSLANTR, 6X,4HV(2),6X,13HV(11)*C-W(60),4X,
      111HV(10)-W(60),6X,7HANGLE E,8X,6HDELT-E,7X,7HANGLE A,7X,6HDELT-A)
 30    II = II + 1
 31    WRITE OUTPUT TAPE 10,32,IN,W(60),V(2),W(83),W(82),W(62),W(81),W(61
      1),W(80)
 32    FORMAT(1H ,I4,1P8E14.7)
       IF(II-50)40,33,33
 33    II = 0
 40    RETURN
       END(0,1,0,1,0,0,0,0,0,0,0,0,0,0,0)
```

```
                 STORAGE LOCATIONS FOR VARIABLES APPEARING IN COMMON STATEMENTS

      DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT
    G 32165 76645        N 32188 76674   RECORD 32561 77461        V 32549 77445        W 32438 77266
   XN 32187 76673       YN 32180 76664       ZN 32173 76655

        EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

 EFN  IFN  LOC     EFN  IFN  LOC     EFN  IFN  LOC     EFN  IFN  LOC     EFN  IFN  LOC
   1    1 00000      9    5 00000     50    6 00000     11    7 00000     12    8 00000
  32    9 00000      3   13 00016      4   14 00020      5   17 00047      6   18 00052
   7   23 00074      7   24 00077      9   26 00124      9   40 00163     10   42 00167
  10   46 00177     30   53 00215     31   54 00220     31   55 00223     33   57 00253
  40   58 00255

                                 STORAGE NOT USED BY PROGRAM

      DEC   OCT            DEC   OCT
      234 00352          32156 76634

                           LOCATIONS OF NAMES IN TRANSFER VECTOR

        DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT
RCOORD    0 00000     (FIL)    2 00002     (STH)    1 00001

     STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION, EQUIVALENCE OR COMMON SENTENCES

        DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT
    IN  233 00351       II   232 00350      TE1  231 00347      TE2  230 00346      TE3  229 00345

                 STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

        DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT            DEC   OCT
8)11    222 00336   8)10     197 00305   8)C      218 00332   8)B      221 00335   8)8      225 00341
1)      226 00342   2)       177 00261   3)       186 00272   6)       190 00276

                    ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY.
RCOORD       (STH)        (FIL)
```

```
00000 RCOORD BCD 1RCOORD
00001 (STH)  BCD 1(STH)
00002 (FIL)  BCD 1(FIL)
00003 S      HTR
00004        HTR
00005        HTR
00006        BCD 1OUTPUT
00007        SXD S,1
00010        SXD S+1,2
00011        SXD S+2,4
       11A   BSS
00012        TSX RCOORD,4
00013 12A    CLA W-69
00014 12A1   TZE 13A
00015        TPL 14A
00016 13A    CLA 2)
00017        STO II
00020 14A    CLA W-69
00021        FAD 3)
00022        STO W-69
00023 15A    CLA W-69
00024        UFA 6)
00025        LRS
00026        ANA 6)+1
00027        LLS
00030        ALS 18
00031        STO IN
00032 16A    LDQ W-1
00033        FMP W-1
00034        STO 1)+1
00035        LDQ W
00036        FMP W
00037        FSB 1)+1
00040        STO 1)+2
00041        LDQ W-1
00042        FMP 3)+1
00043        XCA
00044        FMP W
00045        FDP 1)+2
00046        STO W-58
00047 17A    LDQ V-4
00050        FMP 3)+2
00051        STO TE1
00052 18A    LDQ V-5
00053        FMP 3)+2
00054        STO TE2
00055 19A    CLA V-3
00056        FSB W-18
00057        STO TE3
00060 20A    LDQ 3)+3
00061        FMP V-10
00062        FSB W-54
00063        STO W-55
00064 21A    CLA W-16
00065        FSB W-56
00066        XCA
00067        FMP 3)+2
00070        STO W-57

00071 22A    LDQ W-56
00072        FMP 3)+2
00073        STO W-56
00074 23A    CAL 2)+2
             BSS
00075        TSX (STH),4
00076        PZE 8)8
00077 24A    LDQ W-69
00100        STR
00101        LDQ V-1
00102        STR
00103        LDQ V-2
00104        STR
00105        LDQ TE3
00106        STR
00107        LDQ TE1
00110        STR
00111        LDQ TE2
00112        STR
00113        LDQ V-6
00114        STR
00115        LDQ V-7
00116        STR
             BSS
00117        TSX (FIL),4
00120 25A    CAL 2)+2
             BSS
00121        TSX (STH),4
00122        PZE 8)1I
             BSS
00123        TSX (FIL),4
00124 26A    CAL 2)+2
             BSS
00125        TSX (STH),4
00126        PZE 8)8
00127 27A    LXD 2)+6,1
00130 28A    LDQ V+1,1
00131        STR
00132 28A1   TXI *+1,1,1
00133 28A2   TXL 28A,1,21
00134 30A    LXD 2)+4,1
00135 31A    LDQ W+1,1
00136        STR
00137 31A1   TXI *+1,1,1
00140 31A2   TXL 31A,1,11
00141 33A    LDQ W-27
00142        STR
00143        LDQ W-28
00144        STR
00145        LDQ W-37
00146        STR
00147        LDQ W-38
00150        STR
00151 34A    LXD 2)+7,1
00152 35A    LDQ W+1,1
00153        STR
00154 35A1   TXI *+1,1,1
00155 35A2   TXL 35A,1,70

00156 37A    LXD 2)+8,1
00157 38A    LDQ W+1,1
00160        STR
00161 38A1   TXI *+1,1,1
00162 38A2   TXL 38A,1,90
      40A    BSS
00163        TSX (FIL),4
00164 41A    CLA II
00165 41A1   TZE 42A
00166        TPL 53A
00167 42A    CAL 2)+3
             BSS
00170        TSX (STH),4
00171        PZE 8)B
00172 43A    LXD 2)+4,1
00173 44A    LDQ RECORD+1,1
00174        STR
00175 44A1   TXI *+1,1,1
00176 44A2   TXL 44A,1,12
      46A    BSS
00177        TSX (FIL),4
00200 47A    CAL 2)+2
             BSS
00201        TSX (STH),4
00202        PZE 8)8
00203 48A    LXD 2)+4,1
00204 49A    LDQ RECORD+1,1
00205        STR
00206 49A1   TXI *+1,1,1
00207 49A2   TXL 49A,1,12
      51A    BSS
00210        TSX (FIL),4
00211 52A    CAL 2)+3
             BSS
00212        TSX (STH),4
00213        PZE 8)C
             BSS
00214        TSX (FIL),4
00215 53A    CLA II
00216        ADD 2)+4
00217        STO II
00220 54A    CAL 2)+3
             BSS
00221        TSX (STH),4
00222        PZE 8)10
00223 55A    LDQ IN
00224        STR
00225        LDQ W-59
00226        STR
00227        LDQ V-1
00230        STR
00231        LDQ W-82
00232        STR
00233        LDQ W-81
00234        STR
00235        LDQ W-61
00236        STR
00237        LDQ W-80
```

```
00240          STR
00241          LDQ W-60
00242          STR
00243          LDQ W-79
00244          STR
               BSS
00245          TSX (FIL),4
00246 56A      CLA II
00247          SUB 2)+5
00250 56A1     TZE 57A
00251          TPL 57A
00252          TRA 58A
00253 57A      CLA 2)
00254          STO II
00255 58A      LXD $,1
00256          LXD $+1,2
00257          LXD $+2,4
00260          TRA 1,4
00261 2)       OCT +000000000000
00262          OCT +000002000000
00263          OCT +000006000000
00264          OCT +000012000000
00265          OCT +000001000000
00266          OCT +000062000000
00267          OCT +000011000000
00270          OCT +000067000000
00271          OCT +000120000000
00272 3)       OCT +201400000000
00273          OCT +202400000000
00274          OCT +206712273407
00275          OCT +223444607400
00276 6)       OCT +233000000000
00277          OCT +000000077777
00300          OCT +000000000000
00301          OCT +000001000000
00302          OCT +000000000000
00303          BCD 114.7)
00304          BCD 14,1P8E
00305 8)10     BCD 1(1H ,I
00306          BCD 1LT-A)
00307          BCD 1X,6HDE
00310          BCD 1LE A,7
00311          BCD 1'7HANG
00312          BCD 1T-E,7X
00313          BCD 1,6HDEL
00314          BCD 1E E,8X
00315          BCD 17HANGL
00316          BCD 10),6X,
00317          BCD 10)-W(6
00320          BCD 111HV(1
00321          BCD 1X,
00322          BCD 1(60),4
00323          BCD 11)*C-W
00324          BCD 113HV(1
00325          BCD 12),6X,
00326          BCD 1X,4HV(
00327          BCD 1NTR, 6
00330          BCD 1,6HSLA

00331          BCD 1 IN,5X
00332 8)C      BCD 1 (5HO
00333          BCD 16)
00334          BCD 13X,12A
00335 8)B      BCD 1(1H1,2
00336 8)1I     BCD 1(1H )
00337          BCD 114.7)
00340          BCD 1  1P8E
00341 8)8      BCD 1 (1H ,
```

P. INPUT-OUTPUT

Table 4 illustrates the format that is necessary for the input data. Tables 5 and 6 illustrate the format of the output data that is currently obtained by use of Subroutines OUTONE and OUTPUT. These two subroutines can be easily modified. As they are presented in this report their primary purpose was to collect "debugging" data.

```
*       DATA
SIMPLE SPHERE   10**33/R**12   Z = 0.0   Y VERY SMALL   FEB. 3,1961 E=26DG
    100     66
   1+1.0000000E+00
   2+0.0000000E+00
   3 6.2800000E 09
  13 0.0000000E 00
  14+2.6390000E+02
  15 9.0000000E 01
  16 4.1300000E 01
  17 2.6000000E 01
  18 0.0000000E 00
  19 6.3569117E 03
  20 1.0000000E 00
  21 4.5500000E 01
  22+2.6390000E+02
  23 3.0000000E 02
  24 1.0000000E 00
  25 5.4877431E 01
  26 0.0000000E 00
  27 0.0000000E 00
  34 1.0000000E 33
  35 1.2000000E 01
  36 0.0000000E 00
  40 1.2000000E 03
  41 0.0000000E 00
  42 1.0000000E-05
  43 5.0000000E 01
  44 1.0000000E 00
  45 0.0000000E 00
  46 0.0000000E 00
  47 5.0000000E-01
  48 0.0000000E 00
  49 1.0000000E 00
  50 0.0000000E 00
  51 0.0000000E 00
  63 3.0000000E 00
  68-1.0000000E 00
  69 1.0000000E-04
  71 0.0000000E 00
  72 0.0000000E 00
  73 0.0000000E 00
  74 0.0000000E 00
  75 0.0000000E 00
  76 0.0000000E 00
 -11 0.0000000E 00
SIMPLE SPHERE   10**33/R**12   Z = 0.0   Y VERY SMALL   FEB. 3,1961 E=33DG
    100     66
  17 3.3000000E 01
 -11 0.0000000E 00
SIMPLE SPHERE   10**33/R**12   Z = 0.0   Y VERY SMALL   FEB. 3,1961 E=36DG
    100     66
  17 3.6000000E 01
 -11 0.0000000E 00
SIMPLE SPHERE   10**33/R**12   Z = 0.0   Y VERY SMALL   FEB. 3,1961 E=37DG
    100     66
  17 3.7000000E 01
 -11 0.0000000E 00
```

Table 4. Input Data for a Spherical Ionosphere.

```
 ORDINARY RAY       101SIMPLE SPHERE  10**33/R**12   Z = 0.0   Y VERY SMALL  FEB. 3.1961 E=26DG
1.0000000E 00 0.          6.2800000E 09 0.            0.            0.            0.            0.
0.            0.          0.            0.            0.            4.6059238E 00 1.5707963E 00 7.2082097E-01
4.5378560E-01 0.          6.3569112E 03 1.0000000E 00 7.9412480E-01 4.6059238E 00 3.0000000E 02 1.0000000E 00
5.4677431E 01 0.          0.            0.            0.            6.6569111E 03-4.9581701E 02-4.6394721E 03
4.7480446E 03 1.0000000E 33 1.2000000E 01 0.          0.            0.            0.            1.2000000E 03
0.            9.9999998E-06 5.0000000E 01 1.0000000E 00 0.          0.            5.0000000E-01 0.
1.0000000E 00 0.          0.            -5.0742736E 02-4.7486789E 03 4.1955717E 03 0.            0.
0.            0.          0.            0.            0.            0.            3.0000000E 00 0.
0.            0.          1.0000000E 00-1.0000000E 00 9.9999999E-05 0.           0.            2.2802051E 00
1.0000000E 00 6.3579108E 03 8.4965302E-01 4.6059238E 00 4.3866078E-01-3.9865271E-01-2.3434196E-08 2.2802051E 00
7.0006837E-03 0.          0.            0.            0.            0.            0.            0.
0.            0.          0.
1.0000000E 00 3.2802051E 00 1.0000000E 00 1.4383545E 00 4.8673434E 01 2.6389999E 02 4.3878778E-01-8.9859070E-01
-2.3435490E-08 3.2802051E 00 1.0934017E-05 0.           4.3878779E-01-1.4132452E-04-4.9081066E-12 1.2699290E-04
6.2011474E-05-1.2951176E-12 1.0000000E 00 3.3333333E-06 0.           9.9999998E-01-0.              6.2800000E 09
-5.0437822E-19-2.4659421E-19 8.2772176E-28-4.5618969E-13 5.0024484E-09-1.3781570E-15-5.6196687E-19 1.3516441E-20
5.6001182E 02-5.6324148E-01 5.7858963E 04 1.0510761E 00 3.2802163E 00-2.2964180E-03 2.5994452E 01 5.5473943E-03
-0.           3.2802163E 00 0.          2.5994864E 01 3.0000000E 00 0.             1.3267062E-08 6.4883252E-11
1.0000000E 00-1.0000000E 00 9.9999999E-05 1.0000000E 00 0.            5.1358753E-03-1.1175871E-05-2.2964180E-03
0.            0.          0.            0.            0.            0.            0.
```

Table 5-1. Output Data for "Debugging" Purposes from Tape 6.

```
                    SIMPLE SPHERE  10**33/R**12   Z = 0.0   Y VERY SMALL  FEB. 3,1961 E=2600
 2.0000000E 00 4.2802051E 00 1.0000000E 00 1.8771973E 00 4.8665336E 01 2.6389999E 02 4.3891476E-01-8.9852868E-01

-2.3436785E-08 4.2802051E 00 1.4267350E-05 0.            4.3891478E-01-1.4130502E-04-4.9086491E-12 1.2696661E-04
 6.2020858E-05-1.2952606E-12 1.0000000E 00 3.3333333E-06 0.            9.9999998E-01-0.            6.2800000E 09
-5.1581012E-19-2.5196378E-19 8.4201920E-28-4.6710138E-13 5.1192869E-09-1.3804648E-15-5.7406082E-19 1.3808698E-20
 5.5901442E 02-5.6334660E-01 5.8481141E 04 1.0738028E 00 4.2801423E 00-2.9190779E-03 2.5996310E 01 3.6895828E-03
-0.            4.2801423E 00 0.            2.5995629E 01 3.0000000E 00 0.            1.3265523E-08 8.6718626E-11
 1.0000000E 00-1.0000000E 00 9.9999999E-05 2.0000000E 00 0.            4.3706807E-03 6.2823293E-05-2.9190779E-03
 0.            0.            0.            0.            0.            0.            0.
 3.0000000E 00 5.2802051E 00 1.0000000E 00 2.3161621E 00 4.8657241E 01 2.6389999E 02 4.3904172E-01-8.9846666E-01

-2.3438080E-08 5.2802051E 00 1.7600684E-05 0.            4.3904173E-01-1.4128551E-04-4.9091917E-12 1.2694032E-04
 6.2030233E-05-1.2954039E-12 1.0000000E 00 3.3333333E-06 0.            9.9999998E-01-0.            6.2800000E 09
-5.2689593E-19-2.5747120E-19 8.5658406E-28-4.7829554E-13 5.2390857E-09-1.4420931E-15-5.8643903E-19 1.4107853E-20
 5.5801687E 02-5.6345218E-01 5.9111220E 04 1.0970659E 00 5.2802448E 00-3.7182570E-03 2.5996529E 01 3.4701630E-03
-0.            5.2802448E 00 0.            2.5996639E 01 3.0000000E 00 0.            1.3263984E-08 8.8597317E-11
 1.0000000E 00-1.0000000E 00 9.9999999E-05 3.0000000E 00 0.            3.3604530E-03-3.9637089E-05-3.7182570E-03
 0.            0.            0.            0.            0.            0.            0.
 4.0000000E 00 6.2802051E 00 1.0000000E 00 2.7552490E 00 4.8649147E 01 2.6389999E 02 4.3916864E-01-8.9840461E-01

-2.3439376E-08 6.2802051E 00 2.0934017E-05 0.            4.3916866E-01-1.4126600E-04-4.9097344E-12 1.2691402E-04
 6.2039598E-05-1.2955476E-12 1.0000000E 00 3.3333333E-06 0.            9.9999998E-01-0.            6.2800000E 09
-5.3823936E-19-2.6310846E-19 8.7141767E-28-4.8977747E-13 5.3618962E-09-3.8206091E-15-5.9910571E-19 1.4414011E-20
 5.5701944E 02-5.6355817E-01 5.9749170E 04 1.1208736E 00 6.2802376E 00-4.4077635E-03 2.5996768E 01 3.2315332E-03
-0.            6.2802376E 00 0.            2.5996760E 01 3.0000000E 00 0.            1.3262444E-08 9.0519986E-11
 1.0000000E 00-1.0000000E 00 9.9999999E-05 4.0000000E 00 0.            3.2392172E-03-3.2484531E-05-4.4077635E-03
 0.            0.            0.            0.            0.            0.            0.
 5.0000000E 00 7.2802051E 00 1.0000000E 00 3.1945190E 00 4.8641053E 01 2.6389999E 02 4.3929555E-01-8.9834257E-01

-2.3440671E-08 7.2802051E 00 2.4267350E-05 0.            4.3929556E-01-1.4124649E-04-4.9102771E-12 1.2688773E-04
 6.2048953E-05-1.2956908E-12 1.0000000E 00 3.3333333E-06 0.            9.9999998E-01-0.            6.2800000E 09
-5.4984790E-19-2.6887931E-19 8.8652668E-28-5.0155580E-13 5.4878119E-09-3.8703605E-15-6.1206927E-19 1.4727375E-20
 5.5602198E 02-5.6366435E-01 6.0395159E 04 1.1452417E 00 7.2802491E 00-5.1159263E-03 2.5997558E 01 2.4420060E-03
-0.            7.2802491E 00 0.            2.5998136E 01 3.0000000E 00 0.            1.3260903E-08 9.2487912E-11
 1.0000000E 00-1.0000000E 00 9.9999999E-05 5.0000000E 00 0.            1.8640013E-03-4.3928623E-05-5.1159263E-03
 0.            0.            0.            0.            0.            0.            0.
 6.0000000E 00 8.2802051E 00 2.0000000E 00 3.6338501E 00 4.8632961E 01 2.6389999E 02 4.3942242E-01-8.9828051E-01

-2.3441967E-08 8.2802051E 00 2.7600683E-05 0.            4.3942243E-01-1.4122698E-04-4.9108201E-12 1.2686144E-04
 6.2058299E-05-1.2958342E-12 1.0000000E 00 3.3333333E-06 0.            9.9999998E-01-0.            6.2800000E 09
-5.6172867E-19-2.7478741E-19 9.0191721E-28-5.1363971E-13 5.6169183E-09-4.0111253E-15-6.2533768E-19 1.5048139E-20
 5.5502456E 02-5.6377108E-01 6.1049324E 04 1.1701852E 00 8.2802662E 00-5.8296919E-03 2.5997531E 01 2.4691134E-03
-0.            8.2802662E 00 0.            2.5997436E 01 3.0000000E 00 0.            1.3259361E-08 9.4502312E-11
 1.0000000E 00-1.0000000E 00 9.9999999E-05 6.0000000E 00 0.            2.5632419E-03-6.1035156E-05-5.8296919E-03
 0.            0.            0.            0.            0.            0.            0.
 7.0000000E 00 1.0280205E 01 2.0000000E 00 4.5129395E 00 4.8616781E 01 2.6389999E 02 4.3967609E-01-8.9815636E-01

-2.3444559E-08 1.0280205E 01 3.4267350E-05 0.            4.3967610E-01-1.4118795E-04-4.9119061E-12 1.2680885E-04
 6.2076964E-05-1.2961202E-12 1.0000000E 00 3.3333333E-06 0.            9.9999998E-01-0.            6.2800000E 09
-5.8633108E-19-2.8702769E-19 9.3355968E-28-5.3875341E-13 5.8850231E-09-1.5936303E-15-6.5281622E-19 1.5712533E-20
 5.5302984E 02-5.6398542E-01 6.2382469E 04 1.2218503E 00 1.0280153E 01-7.1094036E-03 2.5998387E 01 1.6123515E-03
-0.            1.0280153E 01 0.            2.5998160E 01 3.0000000E 00 0.            1.3256276E-08 9.8674704E-11
 1.0000000E 00-1.0000000E 00 9.9999999E-05 7.0000000E 00 0.            1.8394553E-03 5.2571297E-05-7.1094036E-03
 0.            0.            0.            0.            0.            0.            0.
 8.0000000E 00 1.2280205E 01 2.0000000E 00 5.3925781E 00 4.8600603E 01 2.6389999E 02 4.3992965E-01-8.9803221E-01

-2.3447151E-08 1.2280205E 01 4.0934017E-05 0.            4.3992966E-01-1.4114891E-04-4.9129924E-12 1.2675626E-04
 6.2095590E-05-1.2964069E-12 1.0000000E 00 3.3333333E-06 0.            9.9999998E-01-0.            6.2800000E 09
-6.1210802E-19-2.9986062E-19 9.6639700E-28-5.6519361E-13 6.1669803E-09-1.6783162E-15-6.8161036E-19 1.6408873E-20
 5.5103499E 02-5.6420140E-01 6.3749802E 04 1.2759997E 00 1.2280224E 01-8.5737705E-03 2.5998542E 01 1.4576050E-03
-0.            1.2280224E 01 0.            2.5998738E 01 3.0000000E 00 0.            1.3253187E-08 1.0304772E-10
```

Table 5-2. Output Data for "Debugging" Purposes from Tape 6.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3,1961 E=2600

IN	SLANTR	V(2)	V(1)*C-W(60)	V(10)-W(60)	ANGLE E	DELT-E	ANGLE A	DELT-A
1	3.2802163E 00	3.2802051E 00	-2.2964180E-03	-1.1175871E-05	2.5994864E 01	5.1358753E-03	0.	0.
2	4.2801423E 00	4.2802051E 00	-2.9190779E-03	6.2823296E-05	2.5995629E 01	4.3706807E-03	0.	0.
3	5.2802448E 00	5.2802051E 00	-3.7182570E-03	-3.9637089E-05	2.5996639E 01	3.3604530E-03	0.	0.
4	6.2802376E 00	6.2802051E 00	-4.4077635E-03	-3.2484531E-05	2.5996760E 01	3.2392172E-03	0.	0.
5	7.2802491E 00	7.2802051E 00	-5.1159263E-03	-4.3928623E-05	2.5998136E 01	1.8640013E-03	0.	0.
6	8.2802662E 00	8.2802051E 00	-5.8296919E-03	-6.1035156E-05	2.5997436E 01	2.5632419E-03	0.	0.
7	1.0280153E 01	1.0280205E 01	-7.1094036E-03	5.2571297E-05	2.5998160E 01	1.8394553E-03	0.	0.
8	1.2280224E 01	1.2280205E 01	-8.5737705E-03	-1.8477440E-05	2.5998738E 01	1.2614505E-03	0.	0.
9	1.4280193E 01	1.4280205E 01	-9.9369287E-03	1.1801720E-05	2.5998787E 01	1.2123586E-03	0.	0.
10	1.6280214E 01	1.6280205E 01	-1.1350632E-02	-8.5830688E-06	2.5998986E 01	1.0138562E-03	0.	0.
11	1.8280242E 01	1.8280205E 01	-1.2772560E-02	-3.7193298E-05	2.5999053E 01	9.4662153E-04	0.	0.
12	2.0280136E 01	2.0280205E 01	-1.4060020E-02	6.8664551E-05	2.5999174E 01	8.2559911E-04	0.	0.
13	2.4280244E 01	2.4280205E 01	-1.6954660E-02	-3.8862228E-05	2.5999346E 01	6.5377716E-04	0.	0.
14	2.8280242E 01	2.8280205E 01	-1.9739389E-02	-3.6954880E-05	2.5999345E 01	6.5527127E-04	0.	0.
15	3.2280224E 01	3.2280205E 01	-2.2507668E-02	-1.8596649E-05	2.5999445E 01	5.5452598E-04	0.	0.
16	3.6280154E 01	3.6280205E 01	-2.5224686E-02	5.0544739E-05	2.5999454E 01	5.4556136E-04	0.	0.
17	4.0280238E 01	4.0280204E 01	-2.8095245E-02	-3.3378601E-05	2.5999654E 01	3.4556488E-04	0.	0.
18	4.4280252E 01	4.4280205E 01	-3.0897141E-02	-4.8160553E-05	2.5999548E 01	4.5143281E-04	0.	0.
19	5.2280149E 01	5.2280204E 01	-3.6366940E-02	5.5789948E-05	2.5999658E 01	3.4150946E-04	0.	0.
20	6.0280247E 01	6.0280204E 01	-4.2037487E-02	-4.1484833E-05	2.5999701E 01	2.9903422E-04	0.	0.
21	6.8280211E 01	6.8280205E 01	-4.7575951E-02	-6.6757202E-06	2.5999759E 01	2.4097761E-04	0.	0.
22	7.6280181E 01	7.6280205E 01	-5.3120613E-02	2.2888184E-05	2.5999778E 01	2.2176770E-04	0.	0.
23	8.4280162E 01	8.4280205E 01	-5.8672905E-02	4.2915344E-05	2.5999774E 01	2.2560968E-04	0.	0.
24	9.2280126E 01	9.2280204E 01	-6.4210892E-02	7.9154968E-05	2.5999810E 01	1.8932431E-04	0.	0.
25	1.0828019E 02	1.0828020E 02	-7.5426102E-02	1.1444092E-05	2.5999834E 01	1.6605897E-04	0.	0.
26	1.2428027E 02	1.2428020E 02	-8.6650848E-02	-6.6757202E-05	2.5999837E 01	1.6243043E-04	0.	0.
27	1.4028022E 02	1.4028020E 02	-9.7742081E-02	-1.1444092E-05	2.5999853E 01	1.4684907E-04	0.	0.
28	1.5628027E 02	1.5628020E 02	-1.0894012E-01	-6.2942505E-05	2.5999884E 01	1.1589977E-04	0.	0.
29	1.7228023E 02	1.7228020E 02	-1.2004662E-01	-2.4795532E-05	2.5999872E 01	1.2742572E-04	0.	0.
30	1.8828029E 02	1.8828020E 02	-1.3125420E-01	-8.3923340E-05	2.5999889E 01	1.1077713E-04	0.	0.
31	2.2028024E 02	2.2028020E 02	-1.5349960E-01	-3.6239624E-05	2.5999897E 01	1.0287972E-04	0.	0.
32	2.5228023E 02	2.5228020E 02	-1.7578316E-01	-2.6702881E-05	2.5999925E 01	7.4705193E-05	0.	0.
33	2.8428025E 02	2.8428020E 02	-1.9808960E-01	-4.1961670E-05	2.5999924E 01	7.5985854E-05	0.	0.
34	3.1628023E 02	3.1628020E 02	-2.2034073E-01	-1.1444092E-05	2.5999950E 01	5.0159202E-05	0.	0.
35	3.4828026E 02	3.4828020E 02	-2.4254608E-01	1.5258789E-05	2.5999936E 01	6.4246466E-05	0.	0.
36	3.8028060E 02	3.8028020E 02	-2.6443481E-01	4.9591064E-05	2.5999938E 01	6.1685145E-05	0.	0.
37	4.1228375E 02	4.1228020E 02	-2.8361130E-01	3.4332275E-05	2.5999885E 01	1.1461911E-04	0.	0.
38	4.1228375E 02	4.1228020E 02	-2.8361130E-01	3.4332275E-05	2.5999885E 01	1.1461911E-04	0.	0.
39	4.2829058E 02	4.2828020E 02	-2.8804398E-01	-2.2888184E-05	2.5999618E 01	3.8163682E-04	0.	0.
40	4.4431643E 02	4.4428020E 02	-2.7328873E-01	-3.8146973E-06	2.5998635E 01	1.3647572E-03	0.	0.
41	4.6043275E 02	4.6028020E 02	-1.6714096E-01	4.5776367E-05	2.5993755E 01	6.2440735E-03	0.	0.
42	4.1228375E 02	4.1228020E 02	-2.8361130E-01	3.4332275E-05	2.5999885E 01	1.1461911E-04	0.	0.
43	4.2028614E 02	4.2028020E 02	-2.8687668E-01	-3.8146973E-06	2.5999787E 01	2.1258964E-04	0.	0.
44	4.2829058E 02	4.2828020E 02	-2.8806305E-01	-3.0517578E-05	2.5999618E 01	3.8163682E-04	0.	0.
45	4.3629912E 02	4.3628020E 02	-2.8503036E-01	3.8146973E-06	2.5999311E 01	6.8878189E-04	0.	0.
46	4.4431644E 02	4.4428020E 02	-2.7322006E-01	2.6702881E-05	2.5998628E 01	1.3711605E-03	0.	0.
47	4.4431644E 02	4.4428020E 02	-2.7322006E-01	2.6702881E-05	2.5998628E 01	1.3711605E-03	0.	0.
48	4.4833113E 02	4.4828020E 02	-2.6126862E-01	3.4332275E-05	2.5998039E 01	1.9606912E-03	0.	0.
49	4.5235268E 02	4.5228020E 02	-2.4251938E-01	-1.9073486E-05	2.5997151E 01	2.8484023E-03	0.	0.
50	4.5638447E 02	4.5628020E 02	-2.1312332E-01	6.8664551E-05	2.5995794E 01	4.2054754E-03	0.	0.

Table 6-1. Output Data from Tape 10.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3,1961 E=260G

IN	SLANTR	V(2)	V(11)*C-W(60)	V(10)-W(60)	ANGLE E	DELT-E	ANGLE A	DELT-A
51	4.6043246E 02	4.6028020E 02	-1.6741943E-01	6.4849854E-05	2.5993686E 01	6.3136561E-03	0.	0.
52	4.6450579E 02	4.6428020E 02	-9.5439911E-02	1.4495850E-04	2.5990346E 01	9.6538320E-03	0.	0.
53	4.6450579E 02	4.6428020E 02	-9.5439911E-02	1.4495850E-04	2.5990346E 01	9.6538320E-03	0.	0.
54	4.6655646E 02	4.6628020E 02	-4.4773102E-02	2.2888184E-04	2.5987965E 01	1.2034366E-02	0.	0.
55	4.6862007E 02	4.6828020E 02	1.9802094E-02	4.2724609E-04	2.5984883E 01	1.5116489E-02	0.	0.
56	4.7070059E 02	4.7028020E 02	1.0268402E-01	7.2479248E-04	2.5980896E 01	1.9103399E-02	0.	0.
57	4.7280349E 02	4.7228020E 02	2.1018982E-01	1.1177063E-03	2.5975617E 01	2.4382921E-02	0.	0.
58	4.7493609E 02	4.7428020E 02	3.5182571E-01	1.9416809E-03	2.5968559E 01	3.1440641E-02	0.	0.
59	4.7710944E 02	4.7628020E 02	5.4186630E-01	3.4027100E-03	2.5958918E 01	4.1081667E-02	0.	0.
60	4.7710944E 02	4.7628020E 02	5.4186630E-01	3.4027100E-03	2.5958918E 01	4.1081667E-02	0.	0.
61	4.7821606E 02	4.7728020E 02	6.6185760E-01	4.6882629E-03	2.5952733E 01	4.7266189E-02	0.	0.
62	4.7933971E 02	4.7828020E 02	8.0370712E-01	6.3629150E-03	2.5945338E 01	5.4661363E-02	0.	0.
63	4.8048387E 02	4.7928020E 02	9.7332001E-01	8.7776184E-03	2.5936385E 01	6.3614247E-02	0.	0.
64	4.8165315E 02	4.8028020E 02	1.1788902E 00	1.2409210E-02	2.5925401E 01	7.4598472E-02	0.	0.
65	4.8285367E 02	4.8128020E 02	1.4322815E 00	1.8035889E-02	2.5911684E 01	8.8315413E-02	0.	0.
66	4.8409396E 02	4.8228020E 02	1.7513924E 00	2.6840210E-02	2.5894148E 01	1.0585171E-01	0.	0.
67	4.8409396E 02	4.8228020E 02	1.7513924E 00	2.6840210E-02	2.5894148E 01	1.0585171E-01	0.	0.
68	4.8473242E 02	4.8278020E 02	1.9440346E 00	3.3222198E-02	2.5883437E 01	1.1656273E-01	0.	0.
69	4.8538543E 02	4.8328020E 02	2.1655006E 00	4.1664124E-02	2.5871027E 01	1.2897275E-01	0.	0.
70	4.8605522E 02	4.8378020E 02	2.4229546E 00	5.2982330E-02	2.5856449E 01	1.4355073E-01	0.	0.
71	4.8674426E 02	4.8428020E 02	2.7265320E 00	6.8634033E-02	2.5839092E 01	1.6090730E-01	0.	0.
72	4.8745552E 02	4.8478020E 02	3.0903854E 00	9.0740204E-02	2.5818072E 01	1.8192742E-01	0.	0.
73	4.8819146E 02	4.8528020E 02	3.5360603E 00	1.2361526E-01	2.5792013E 01	2.0798694E-01	0.	0.
74	4.8895369E 02	4.8578020E 02	4.0955658E 00	1.7445755E-01	2.5758884E 01	2.4111571E-01	0.	0.
75	4.8973798E 02	4.8628020E 02	4.8191681E 00	2.5856400E-01	2.5715396E 01	2.8460416E-01	0.	0.
76	4.8973798E 02	4.8628020E 02	4.8191681E 00	2.5856400E-01	2.5715396E 01	2.8460416E-01	0.	0.
77	4.9013373E 02	4.8653020E 02	5.2637558E 00	3.2129669E-01	2.5688341E 01	3.1165854E-01	0.	0.
78	4.9052556E 02	4.8678020E 02	5.7776680E 00	4.0545654E-01	2.5656798E 01	3.4320142E-01	0.	0.
79	4.9090439E 02	4.8703020E 02	6.3705139E 00	5.1927185E-01	2.5620041E 01	3.7995808E-01	0.	0.
80	4.9125068E 02	4.8728020E 02	7.0464859E 00	6.7227936E-01	2.5577706E 01	4.2229416E-01	0.	0.
81	4.9157565E 02	4.8753020E 02	7.8001518E 00	8.7360382E-01	2.5529988E 01	4.7001114E-01	0.	0.
82	4.9184014E 02	4.8778020E 02	8.6133842E 00	1.1283607E 00	2.5477955E 01	5.2204480E-01	0.	0.
83	4.9184014E 02	4.8778020E 02	8.6133842E 00	1.1283607E 00	2.5477955E 01	5.2204480E-01	0.	0.
84	4.9195053E 02	4.8790520E 02	9.0341530E 00	1.2757263E 00	2.5450830E 01	5.4917005E-01	0.	0.
85	4.9204627E 02	4.8803020E 02	9.4594002E 00	1.4352684E 00	2.5423270E 01	5.7672985E-01	0.	0.
86	4.9212759E 02	4.8815520E 02	9.8860130E 00	1.6060143E 00	2.5395513E 01	6.0448710E-01	0.	0.
87	4.9219559E 02	4.8828020E 02	1.0310882E 01	1.7863007E 00	2.5367702E 01	6.3229750E-01	0.	0.
88	4.9225110E 02	4.8840520E 02	1.0731991E 01	1.9749641E 00	2.5340025E 01	6.5997450E-01	0.	0.
89	4.9229532E 02	4.8853020E 02	1.1147602E 01	2.1705627E 00	2.5312591E 01	6.8740838E-01	0.	0.
90	4.9232947E 02	4.8865520E 02	1.1556541E 01	2.3718376E 00	2.5285490E 01	7.1450992E-01	0.	0.
91	4.9235436E 02	4.8878020E 02	1.1958362E 01	2.5779800E 00	2.5258763E 01	7.4123710E-01	0.	0.
92	4.9237142E 02	4.8890520E 02	1.2352257E 01	2.7876472E 00	2.5232448E 01	7.6755168E-01	0.	0.
93	4.9238144E 02	4.8903020E 02	1.2738235E 01	3.0001869E 00	2.5206582E 01	7.9341803E-01	0.	0.
94	4.9238521E 02	4.8915520E 02	1.3116363E 01	3.2149734E 00	2.5181148E 01	8.1885195E-01	0.	0.
95	4.9238352E 02	4.8928020E 02	1.3486748E 01	3.4314194E 00	2.5156167E 01	8.4383250E-01	0.	0.
96	4.9237704E 02	4.8940520E 02	1.3849567E 01	3.6490135E 00	2.5131609E 01	8.6839045E-01	0.	0.
97	4.9235157E 02	4.8965520E 02	1.4554005E 01	4.0866089E 00	2.5083744E 01	9.1625578E-01	0.	0.
98	4.9231305E 02	4.8990520E 02	1.5231567E 01	4.5249557E 00	2.5037438E 01	9.6256191E-01	0.	0.
99	4.9226385E 02	4.9015520E 02	1.5885216E 01	4.9630165E 00	2.4992545E 01	1.0074546E 00	0.	0.
100	4.9220610E 02	4.9040520E 02	1.6517315E 01	5.3998146E 00	2.4948932E 01	1.0510677E 00	0.	0.

Table 6-2. Output Data from Tape 10.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 Y VERY SMALL FEB. 3.1961 E=26DG

IN	SLANTR	V(2)	V(11)*C-W(60)	V(10)-W(60)	ANGLE E	DELT-E	ANGLE A	DELT-A
101	4.9214146E 02	4.9065520E 02	1.7130016E 01	5.3347359E 00	2.4906487E 01	1.0935124E 00	0.	0.
102	4.9207153E 02	4.9090520E 02	1.7724888E 01	6.2670479E 00	2.4865093E 01	1.1349068E 00	0.	0.
103	4.9199687E 02	4.9115520E 02	1.8304085E 01	6.6969757E 00	2.4824646E 01	1.1753520E 00	0.	0.
104	4.9183705E 02	4.9165520E 02	1.9420559E 01	7.5466183E 00	2.4746254E 01	1.2537408E 00	0.	0.
105	4.9166636E 02	4.9215520E 02	2.0489605E 01	8.3897247E 00	2.4670725E 01	1.3292748E 00	0.	0.
106	4.9148804E 02	4.9265520E 02	2.1518730E 01	9.2204819E 00	2.4597570E 01	1.4024301E 00	0.	0.
107	4.9130426E 02	4.9315520E 02	2.2513950E 01	1.0041428E 01	2.4526440E 01	1.4735602E 00	0.	0.
108	4.9111628E 02	4.9365520E 02	2.3480266E 01	1.0853451E 01	2.4457020E 01	1.5429799E 00	0.	0.
109	4.9092530E 02	4.9415520E 02	2.4421440E 01	1.1657089E 01	2.4389100E 01	1.6108993E 00	0.	0.
110	4.9073188E 02	4.9465520E 02	2.5340874E 01	1.2453209E 01	2.4322460E 01	1.6775393E 00	0.	0.
111	4.9053696E 02	4.9515520E 02	2.6240871E 01	1.3242081E 01	2.4256945E 01	1.7430547E 00	0.	0.
112	4.9034063E 02	4.9565520E 02	2.7123955E 01	1.4024597E 01	2.4192421E 01	1.8075784E 00	0.	0.
113	4.9014326E 02	4.9615520E 02	2.7991978E 01	1.4801239E 01	2.4128764E 01	1.8712355E 00	0.	0.
114	4.8994538E 02	4.9665520E 02	2.8846248E 01	1.5572197E 01	2.4065884E 01	1.9341151E 00	0.	0.
115	4.8974698E 02	4.9715520E 02	2.9688354E 01	1.6338116E 01	2.4003682E 01	1.9963179E 00	0.	0.
116	4.8954836E 02	4.9765520E 02	3.0519759E 01	1.7099239E 01	2.3942113E 01	2.0578862E 00	0.	0.
117	4.8915055E 02	4.9865520E 02	3.2152355E 01	1.8608814E 01	2.3820574E 01	2.1794254E 00	0.	0.
118	4.8875288E 02	4.9965520E 02	3.3751873E 01	2.0103142E 01	2.3700806E 01	2.2991937E 00	0.	0.
119	4.8835580E 02	5.0065520E 02	3.5323261E 01	2.1584251E 01	2.3582515E 01	2.4174844E 00	0.	0.
120	4.8795963E 02	5.0165520E 02	3.6870712E 01	2.3053741E 01	2.3465406E 01	2.5345933E 00	0.	0.
121	4.8756485E 02	5.0265520E 02	3.8397495E 01	2.4512874E 01	2.3349311E 01	2.6506886E 00	0.	0.
122	4.8717145E 02	5.0365520E 02	3.9906414E 01	2.5962818E 01	2.3234048E 01	2.7659519E 00	0.	0.
123	4.8677946E 02	5.0465520E 02	4.1399868E 01	2.7404682E 01	2.3119443E 01	2.8805563E 00	0.	0.
124	4.8638915E 02	5.0565520E 02	4.2879550E 01	2.8839085E 01	2.3005413E 01	2.9945872E 00	0.	0.
125	4.8600050E 02	5.0665520E 02	4.4347145E 01	3.0266823E 01	2.2891845E 01	3.1081551E 00	0.	0.
126	4.8561365E 02	5.0765520E 02	4.5803897E 01	3.1688389E 01	2.2778679E 01	3.2213204E 00	0.	0.
127	4.8484513E 02	5.0965520E 02	4.8689544E 01	3.4515350E 01	2.2553197E 01	3.4468029E 00	0.	0.
128	4.8408381E 02	5.1165520E 02	5.1543705E 01	3.7323276E 01	2.2328518E 01	3.6714813E 00	0.	0.
129	4.8332977E 02	5.1365520E 02	5.4371769E 01	4.0114696E 01	2.2104323E 01	3.8956767E 00	0.	0.
130	4.8258300E 02	5.1565520E 02	5.7177875E 01	4.2891628E 01	2.1880354E 01	4.1196459E 00	0.	0.
131	4.8184375E 02	5.1765520E 02	5.9964867E 01	4.5655311E 01	2.1656430E 01	4.3435698E 00	0.	0.
132	4.8111172E 02	5.1965520E 02	6.2735428E 01	4.8407218E 01	2.1432406E 01	4.5675936E 00	0.	0.
133	4.8038703E 02	5.2165520E 02	6.5491314E 01	5.1148151E 01	2.1208161E 01	4.7918387E 00	0.	0.
134	4.7966989E 02	5.2365520E 02	6.8233753E 01	5.3878605E 01	2.0983610E 01	5.0163895E 00	0.	0.
135	4.7896019E 02	5.2565520E 02	7.0964004E 01	5.6599250E 01	2.0758706E 01	5.2412938E 00	0.	0.
136	4.7825773E 02	5.2765520E 02	7.3683174E 01	5.9310753E 01	2.0533375E 01	5.4666247E 00	0.	0.
137	4.7687538E 02	5.3165520E 02	7.9090130E 01	6.4706783E 01	2.0081296E 01	5.9187036E 00	0.	0.
138	4.7552298E 02	5.3565520E 02	8.4458717E 01	7.0068665E 01	1.9627151E 01	6.3728482E 00	0.	0.
139	4.7420058E 02	5.3965520E 02	8.9791607E 01	7.5397701E 01	1.9170812E 01	6.8291877E 00	0.	0.
140	4.7290850E 02	5.4365520E 02	9.5090282E 01	8.0694469E 01	1.8712172E 01	7.2878274E 00	9.8911701E-03	-9.8911701E-03
141	4.7164694E 02	5.4765520E 02	1.0035575E 02	8.5959381E 01	1.8251248E 01	7.7467511E 00	0.	0.
142	4.7041619E 02	5.5165520E 02	1.0558853E 02	9.1192539E 01	1.7787991E 01	8.2120084E 00	0.	0.
143	4.6921613E 02	5.5565520E 02	1.1078929E 02	9.6394352E 01	1.7322420E 01	8.6775801E 00	0.	0.
144	4.6804739E 02	5.5965520E 02	1.1595780E 02	1.0156436E 02	1.6854547E 01	9.1454523E 00	0.	0.
145	4.6691003E 02	5.6365520E 02	1.2109425E 02	1.0670266E 02	1.6384413E 01	9.6155864E 00	0.	0.
146	4.6473008E 02	5.7165520E 02	1.3127106E 02	1.1688383E 02	1.5437478E 01	1.0562522E 01	0.	0.
147	4.6267830E 02	5.7965520E 02	1.4131864E 02	1.2693630E 02	1.4481937E 01	1.1518063E 01	0.	0.
148	4.6075640E 02	5.8765520E 02	1.5123575E 02	1.3685859E 02	1.3518227E 01	1.2481773E 01	0.	0.
149	4.5896591E 02	5.9565520E 02	1.6102116E 02	1.4664932E 02	1.2546701E 01	1.3457296E 01	9.8911701E-03	-9.8911701E-03
150	4.5730842E 02	6.0365520E 02	1.7067335E 02	1.5630695E 02	1.1567826E 01	[illegible] 01	0.	0.

Table 6-3. Output Data from Tape 10.

SIMPLE SPHERE 10**33/R**12 Z = 0.0 V VERY SMALL FEB. 3,1961 E=26DG

IN	SLANTR	V(2)	V(11)*C-W(60)	V(13)-W(60)	ANGLE E	DELT-E	ANGLE A	DELT-A
151	4.5578519E 02	6.1165520E 02	1.8019119E 02	1.6583028E 02	1.0582244E 01	1.5417756E 01	0.	0.
152	4.5314751E 02	6.2765520E 02	1.9881790E 02	1.8446804E 02	8.5925636E 00	1.7407436E 01	0.	0.
153	4.5106232E 02	6.4365520E 02	2.1689203E 02	2.0255327E 02	6.5820498E 00	1.9417950E 01	0.	0.
154	4.4953725E 02	6.5965520E 02	2.3440598E 02	2.2007836E 02	4.5553746E 00	2.1444625E 01	0.	0.
155	4.4857310E 02	6.7565520E 02	2.5135401E 02	2.3703752E 02	2.5174736E 00	2.3482526E 01	0.	0.
156	4.4818835E 02	6.9165520E 02	2.6773260E 02	2.5342726E 02	4.7352738E-01	2.5526472E 01	0.	0.
157	4.4836954E 02	7.0765520E 02	2.8354027E 02	2.6924607E 02	1.5715396E 00	2.4428460E 01	0.	0.
158	4.5044008E 02	7.3965520E 02	3.1344744E 02	2.9917554E 02	5.6436183E 00	2.0356381E 01	0.	0.
159	4.5475843E 02	7.7165520E 02	3.4110675E 02	3.2685715E 02	9.6583892E 00	1.6341610E 01	0.	0.
160	4.6126162E 02	8.0365520E 02	3.6658133E 02	3.5235401E 02	1.3578833E 01	1.2421166E 01	0.	0.
161	4.6985884E 02	8.3565520E 02	3.8996181E 02	3.7575679E 02	1.7373080E 01	8.6269197E 00	0.	0.
162	4.8043762E 02	8.6765519E 02	4.1136072E 02	3.9717800E 02	2.1015699E 01	4.9843011E 00	0.	0.
163	4.9287056E 02	8.9965520E 02	4.3090550E 02	4.1674507E 02	2.4488013E 01	1.5119870E 00	9.8911701E-03	-9.8911701E-03
164	5.2274979E 02	9.6365520E 02	4.6498168E 02	4.5086583E 02	3.0879896E 01	-4.8798959E 00	0.	0.
165	5.5839539E 02	1.0276552E 03	4.9329149E 02	4.7922024E 02	3.6519111E 01	-1.0519111E 01	0.	0.
166	5.9877833E 02	1.0916552E 03	5.1686395E 02	5.0283729E 02	4.1440417E 01	-1.5440418E 01	0.	0.
167	6.4300658E 02	1.1556552E 03	5.3659111E 02	5.2260904E 02	4.5712849E 01	-1.9712850E 01	0.	0.
168	6.9034160E 02	1.2196552E 03	5.5321150E 02	5.3927402E 02	4.9417751E 01	-2.3417752E 01	0.	0.
169	7.4018750E 02	1.2836552E 03	5.6732103E 02	5.5342812E 02	5.2635686E 01	-2.6635687E 01	0.	0.
170	8.4561546E 02	1.4116552E 03	5.8930389E 02	5.7600017E 02	5.7894388E 01	-3.1894388E 01	0.	0.
171	9.5658753E 02	1.5396552E 03	6.0676265E 02	5.9304810E 02	6.1961260E 01	-3.5961260E 01	0.	0.
172	1.0713299E 03	1.6676552E 03	6.1991215E 02	6.0628677E 02	6.5170284E 01	-3.9170285E 01	0.	0.
173	1.1887967E 03	1.7956552E 03	6.2035513E 02	6.1681892E 02	6.7751948E 01	-4.1751949E 01	0.	0.
174	1.3092422E 03	1.9236552E 03	6.3382043E 02	6.2537339E 02	6.9865861E 01	-4.3865861E 01	0.	0.

Table 6-4. Output Data from Tape 10

ACKNOWLEDGEMENTS

This problem has been discussed with numerous groups interested in its solution. Special mention should be made of Dr. Milford Davis and Dr. Stanley Greenfield of RAND Corporation with whom countless hours were spent (in the early part of 1959) in mutual search for the best approach to a more general problem of which this one is a subset. During that time discussions with Dr. I. Yabroff and Dr. J. Brandstatter from Stanford Research Institute were also helpful, for they introduced the writer to Haselgrove's published papers. But most of all the writer would like to express his gratitude to Dr. Jennifer Haselgrove, Manchester University, England, for the private communications through which he had access to her then unpublished works.

Because this effort could never have been possible without financial support the writer would like to express his gratefulness to General Electric Company, whose research funds permitted the initiation of this work, and to the government agencies ARPA, through AFCRL, and DASA whose funding has carried this problem to its present state of development.

REFERENCES

1. Synge, J. L. (a) Hamilton's Method in Geometrical Optics, Chapter 2, University of Maryland, Institute of Fluid Dynamics and Applied Mathematics, Lecture Series 9 (1951); (b) Geometrical Mechanics and de Broglie Waves, Chapter 2, Cambridge University Press, London (1954).

2. Haselgrove, J. (a) Ray Theory and a New Method for Ray Tracing, Report on Conference on "Physics of the Ionosphere," p. 355, The Physical Society, London, (1955); (b) Proceedings of Physical Society, London B, LXX, 653; (c) With C. B. Haselgrove, Twisted Ray Paths in the Ionosphere I, preprint of paper to be published and sent in private communication to the writer (1959).

3. Mitra, S. K. The Upper Atmosphere, The Asiatic Society Monograph Series, p. 184.

4. Yabroff, I. Computation of Whistler Ray Paths. SRI Project 2241, Menlo Park, California: Stanford Research Institute, December 1959.

5. Hartree, D. R., Michel, J. G. and Nicolson, P. Report on Conference on "Meteorological Factors in Radio Propagation", London: The Physical Society, p. 161, 1946.

6. Wong, M. S. Proceedings of the Institute of Radio Engineers, New York, p. 1628, 1958.

7. Dudziak, W. F. The Time Dependent Blackout Problem, SP-123 Santa Barbara, California: Technical Military Planning Operation, General Electric Company. 31 March 1961. S/RD

8. Millman, G. H. "The Geometry of the Earth's Magnetic Field at Ionospheric Heights." Journal of Geophysical Research, Vol. 64, p. 717, July 1959.

9. Chapman, S. The Earth's Magnetism. Methuen and Company. 1951.

10. Archer, D. H. Position and Rate Errors Due to Refraction Through Ionized Regions. RM 59TMP-26, Santa Barbara, California: Technical Military Planning Operation, General Electric Company, December 31, 1959.

11. DASA 1229, "Electromagnetic Blackout Guide," Parts I and II. (Effects of High Altitude Nuclear Bursts on Electromagnetic Waves). RM 61TMP-14, Santa Barbara, California: Technical Military Planning Operation, General Electric Company, S/RD

12. Dudziak, W. F. Computer Solution of Transient and Quasi-Equilibrium Atmospheric Deionization Equations. RM 61TMP-33, Santa Barbara, California: Technical Military Planning Operation, General Electric Company, May 1, 1961.

13. Reference Manual 709/7090 FORTRAN Programming System, C28-6054-2, International Business Machines Corporation, White Plains, New York.